STU FEI

Programovacie techniky

3. Zložitosť, O notácia, bubble sort, insertion sort, quick sort, merge sort

Triedenie

Usporiadanie napr. podľa veľkosti

1, 77, 89, 102, 206, 234

„Martin“ < „Alexander“

alebo

„Martin“ > „Alexander“

(M, r): usporiadaj množinu M vzhľadom na reláciu r

Triedenie

- Jednorázové triedenie veľkého množstva údajov
- Utrieďovanie „položka po položke“, okamžite po vzniku daného údaja

Kritéria

Časová zložitosť (time complexity):

- Počet porovnaní
- Počet výmien (swapov)

Priestorová zložitosť (space complexity):

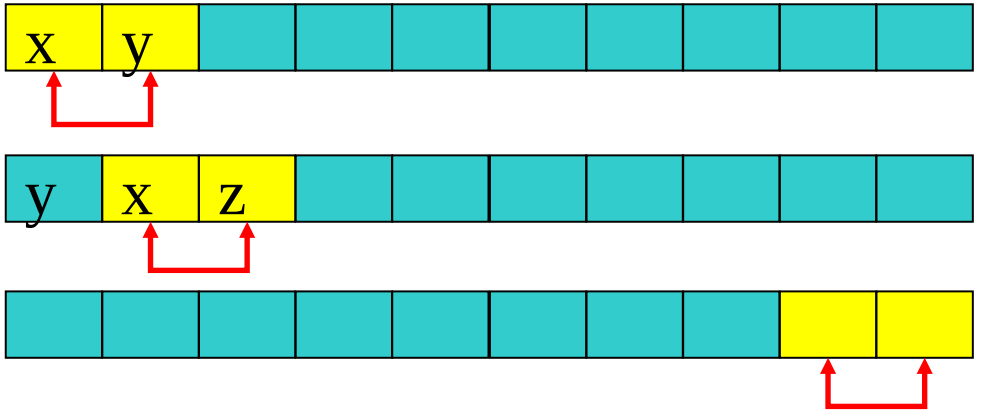
- Koľko pamäte potrebujeme?
- V tom istom poli (in-place), alebo je potrebné ďalšie pomocné pole?

Bubble sort

Idea: pri prechode poľom porovnať dva za sebou idúce prvky poľa. Ak sú v nesprávnom poradí, tak sú vymenené.

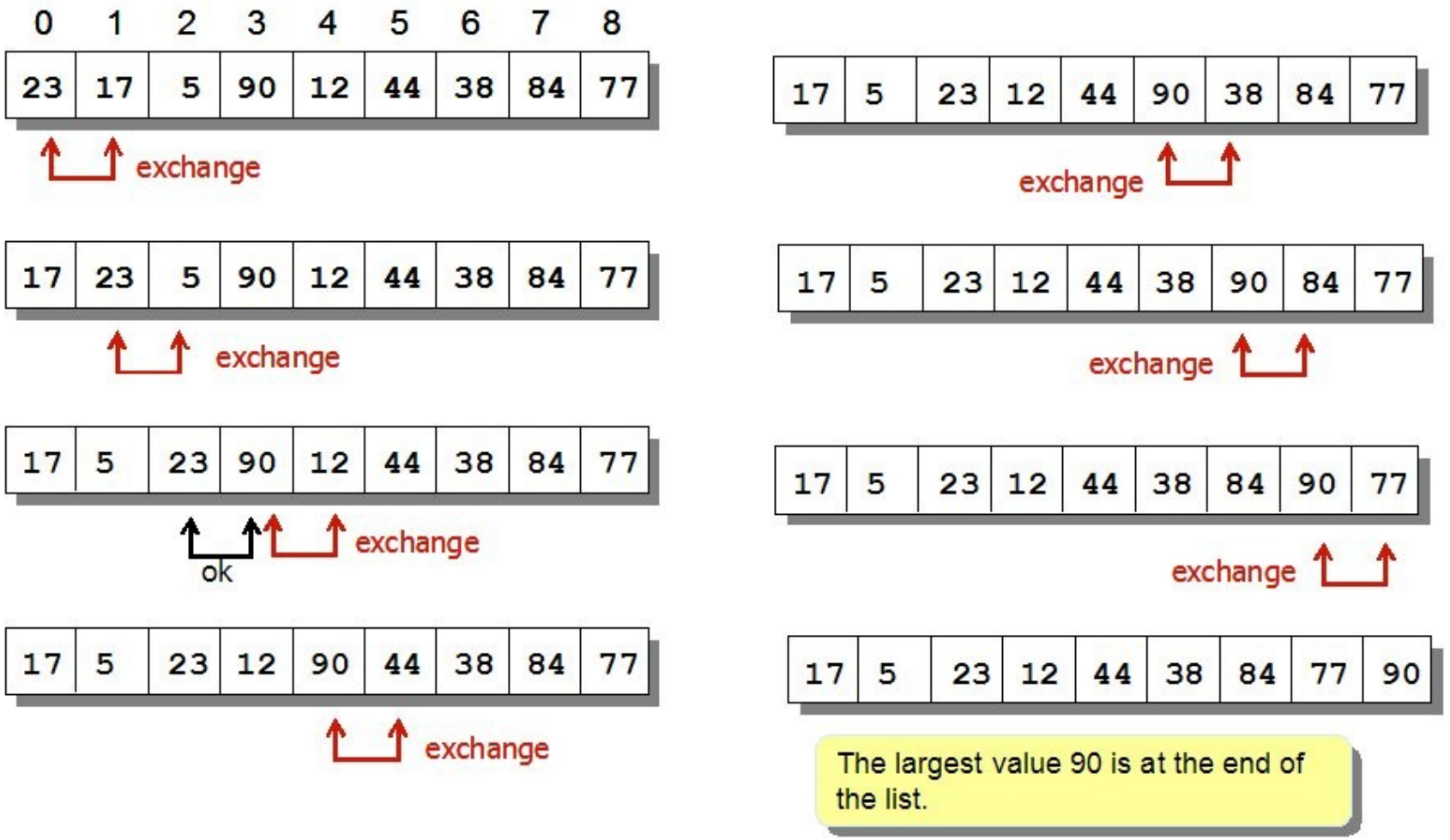
ak x > y

...



Opakovať pokým neutriedime celé pole.

Bubble sort


0 1 2 3 4 5 6 7 8
23 17 5 90 12 44 38 84 77
exchange
17 23 5 90 12 44 38 84 77
exchange
17 5 23 90 12 44 38 84 77
ok
exchange
17 5 23 12 90 44 38 84 77
exchange
17 5 23 12 44 90 38 84 77
exchange
17 5 23 12 44 38 90 84 77
exchange
17 5 23 12 44 38 84 90 77
exchange
17 5 23 12 44 38 84 77 90
The largest value 90 is at the end of the list.

Bubble sort

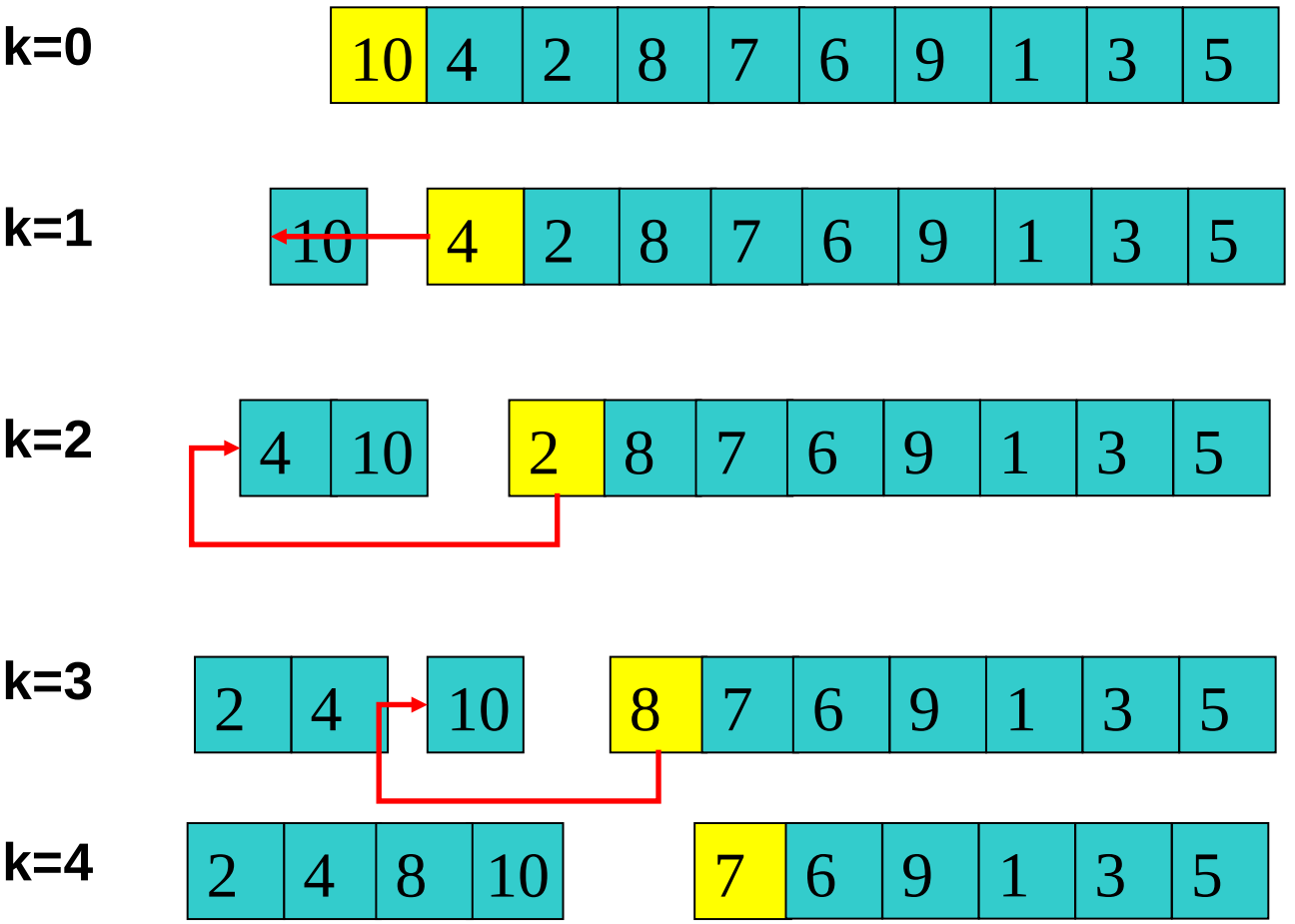
```
void prechod (int *pole, int N) {
    int j;
    for(j=0; j < N-1; j++){              //nie N prečo?
      if(pole[j] < pole[j+1]) {          //porovnanie
        vymen(pole+j, pole+j+1); //výmena
      }
    }
}
```

Insertion sort

V k-tom kroku vybrať k-ty prvok poľa a uložiť ho na svoju pozíciu medzi prvky s indexmi 0,...,k-1.

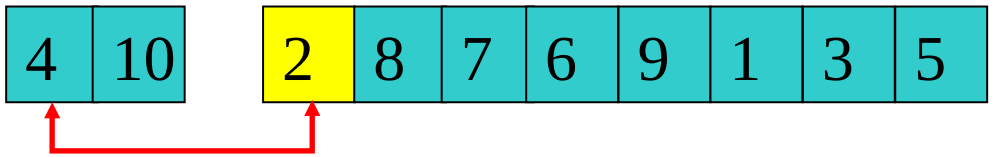
Vkladanie kariet – do už utriedených kariet vlož ďalšiu.

Insertion sort: pole

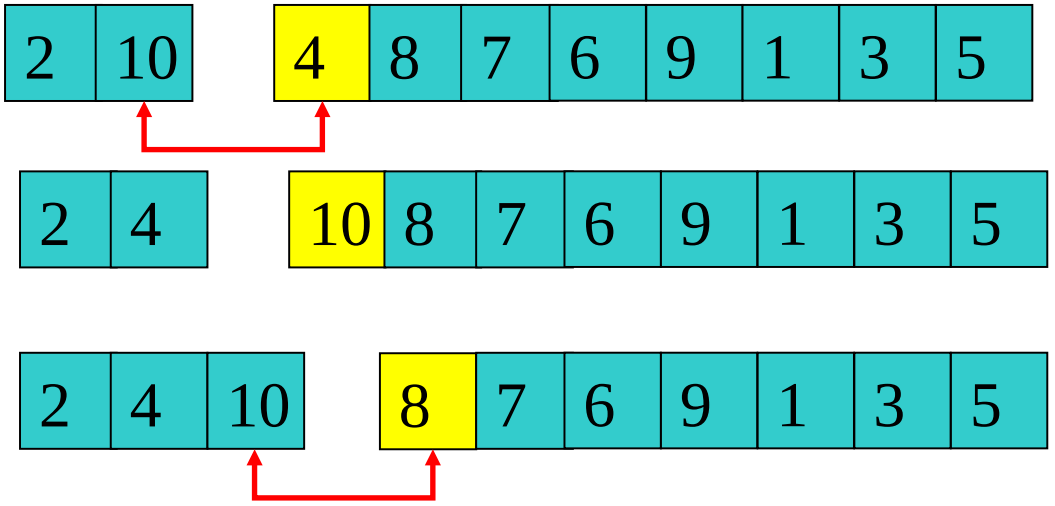


Insertion sort: pomocou výmen

Vloženie na správne miesto možno realizovať postupnými výmenami.



Realizácia umiestenia pomocou výmen:



Insertion sort, bubble sort

Bubble sort: $O(n^2)$

Vonkajší cyklus: (n-1)-krát

Vnútorný cyklus: (n-1) + (n-2) + (n-3) + ... + 1

Insertion sort: $O(n^2)$

Vonkajší cyklus: ?

Vnútorný cyklus: ?

O-notácia zanedbáva konštanty: $3n^2 = O(n^2)$

$n^2 + 100 = O(n^2)$

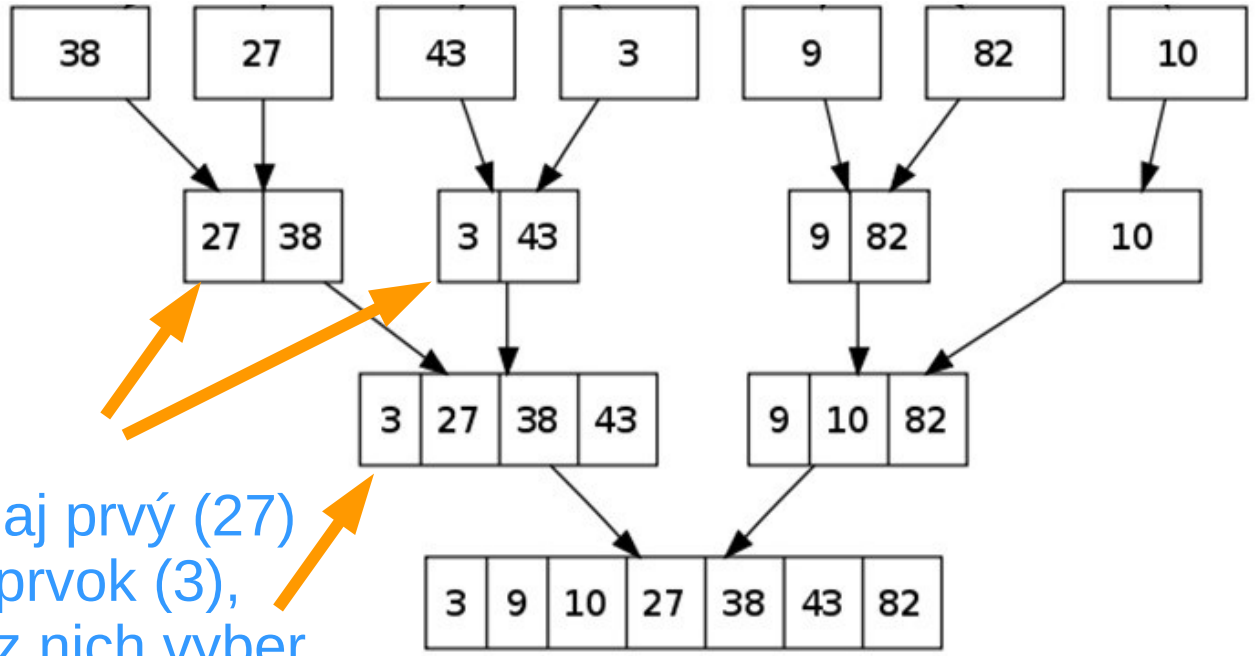
$n^2 / 100 = O(n^2)$

Merge sort

Idea: rozdelím pole na 2 časti. Tie utriedim a spojím.

Ak časť pola neviem roztriediť, pole opätovne rozdelím na 2 časti.

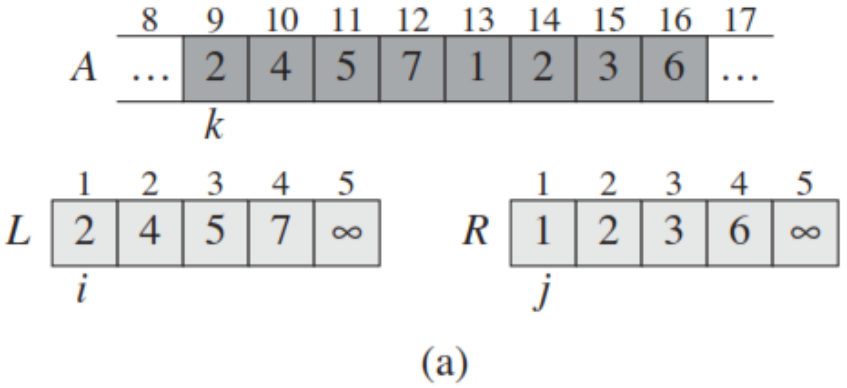
Merge sort: spájanie



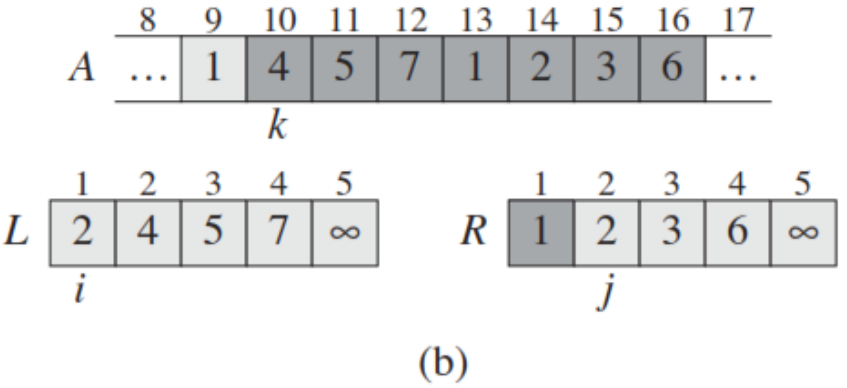
Porovnaj prvý (27) a prvý prvok (3), menší z nich vyber (3) a vlož do spojeného poľa

Obrázok zdroj: http://anandgrathod.blogspot.sk/2013/02/merge-sort.html

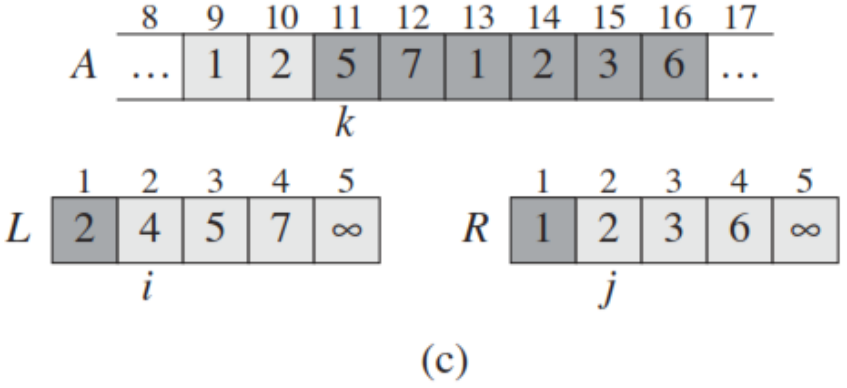
Merge sort: príklad merge



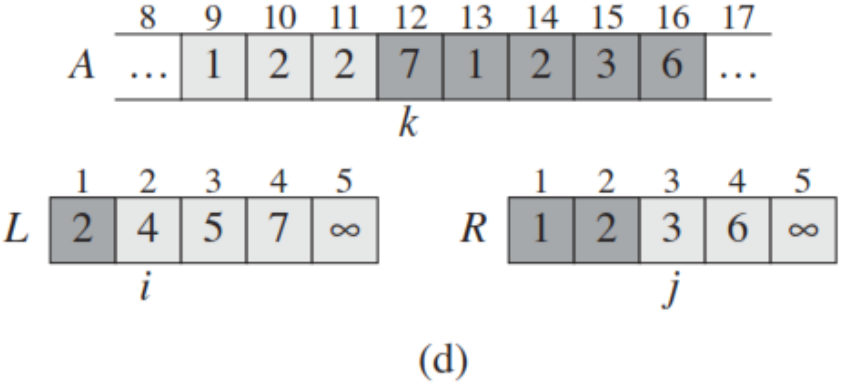
(a)



(b)



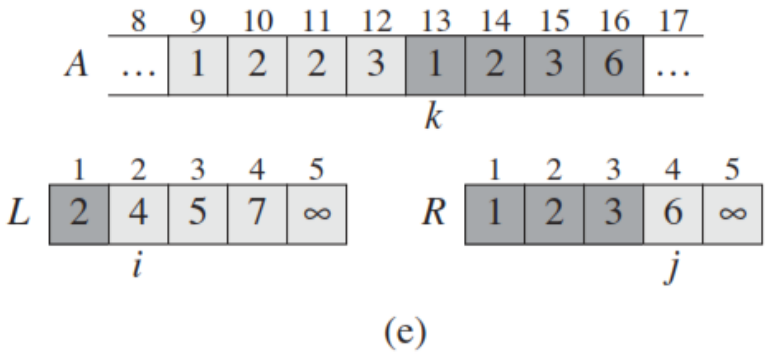
(c)



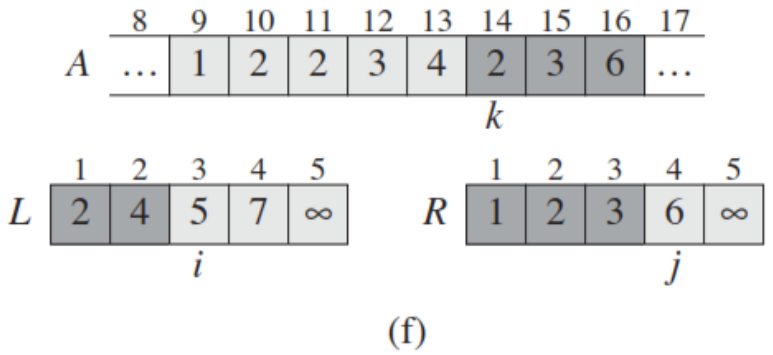
(d)

Merge sort je stabilný triediaci algoritmus

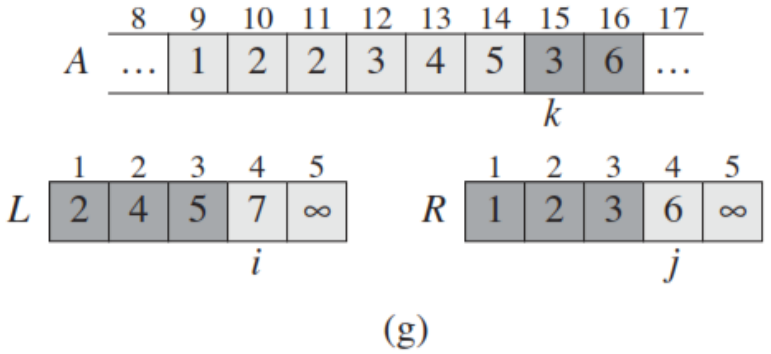
Merge sort: príklad merge



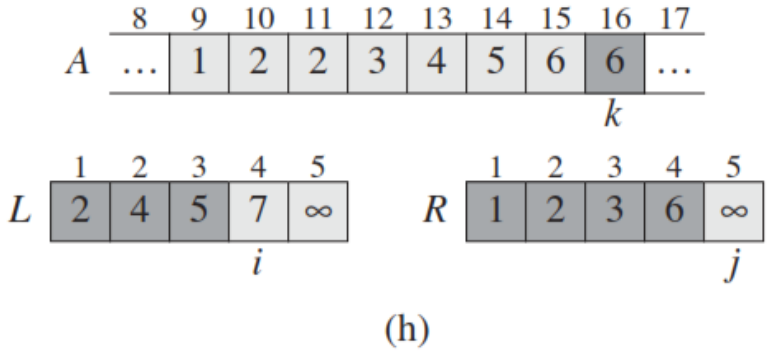
(e)



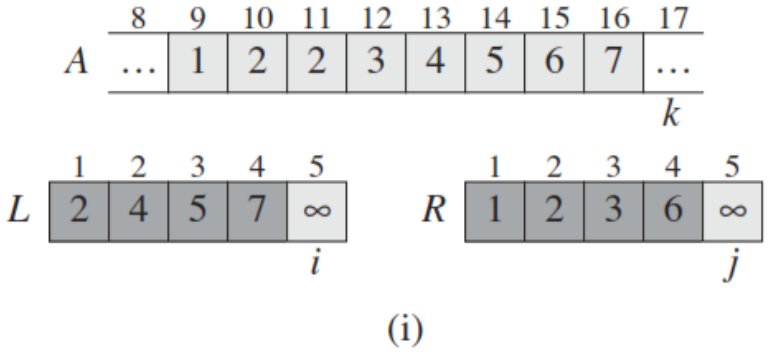
(f)



(g)



(h)



(i)

Merge: pseudokód

A je pole A[p..q..r]

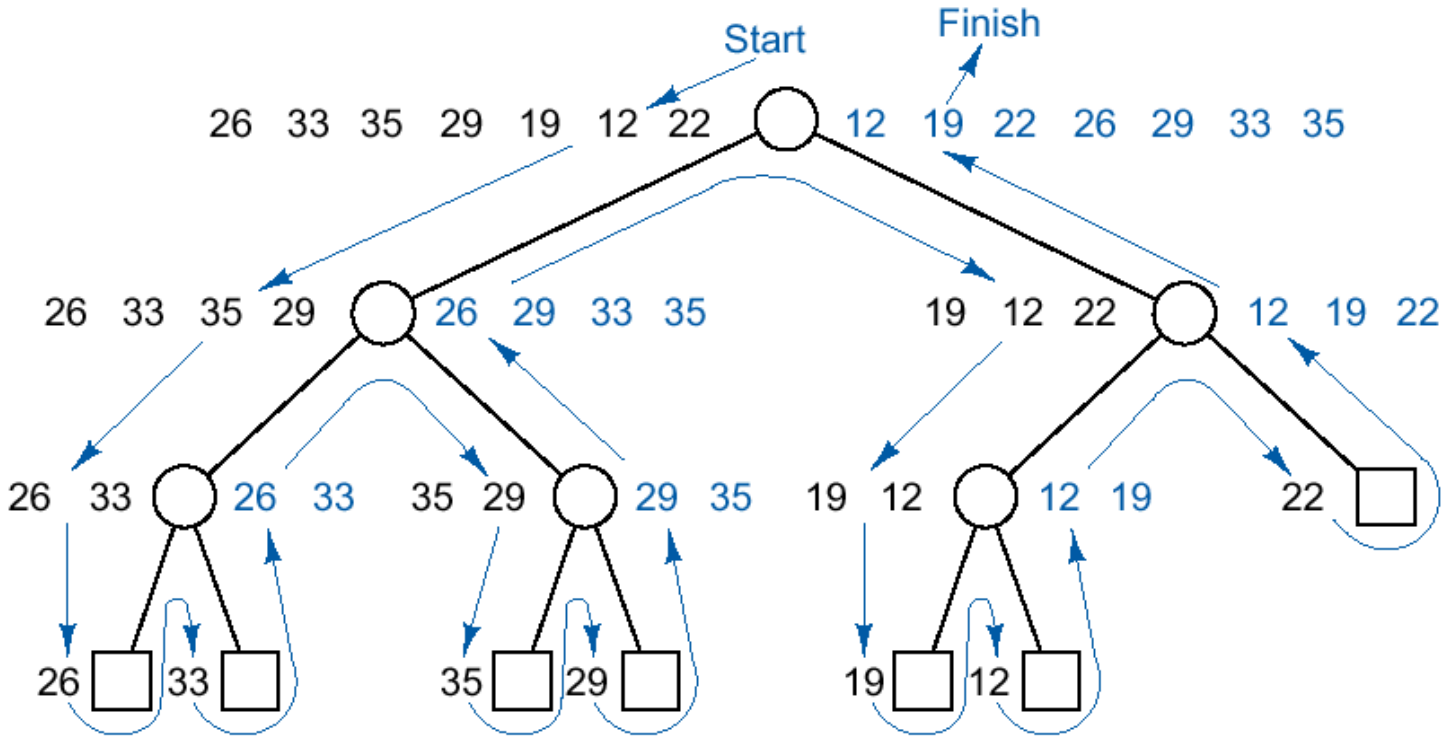
```
MERGE(A, p, q, r)
 1  n1 = q − p + 1                                   Od p po q
 2  n2 = r − q                                       Od q+1 po r
 3  let L[1..n1 + 1] and R[1..n2 + 1] be new arrays  Pomocné polia
 4  for i = 1 to n1
 5      L[i] = A[p + i − 1]                          Prekopíruj do pomocného poľa
 6  for j = 1 to n2
 7      R[j] = A[q + j]                              Prekopíruj do pomocného poľa
 8  L[n1 + 1] = ∞                                    Koniec poľa: nastav na max. celé číslo
 9  R[n2 + 1] = ∞
10  i = 1
11  j = 1
12  for k = p to r
13      if L[i] ≤ R[j]
14          A[k] = L[i]
15          i = i + 1
16      else A[k] = R[j]
17          j = j + 1
```

Pseudokód

Zápis algoritmu pomocou:

for / end for
for each / end for
while / do while / end while
if … else / end if
<, >, = a všetky ostatné matematické symboly
operácie: napr. vymeň(x, y)
input / return

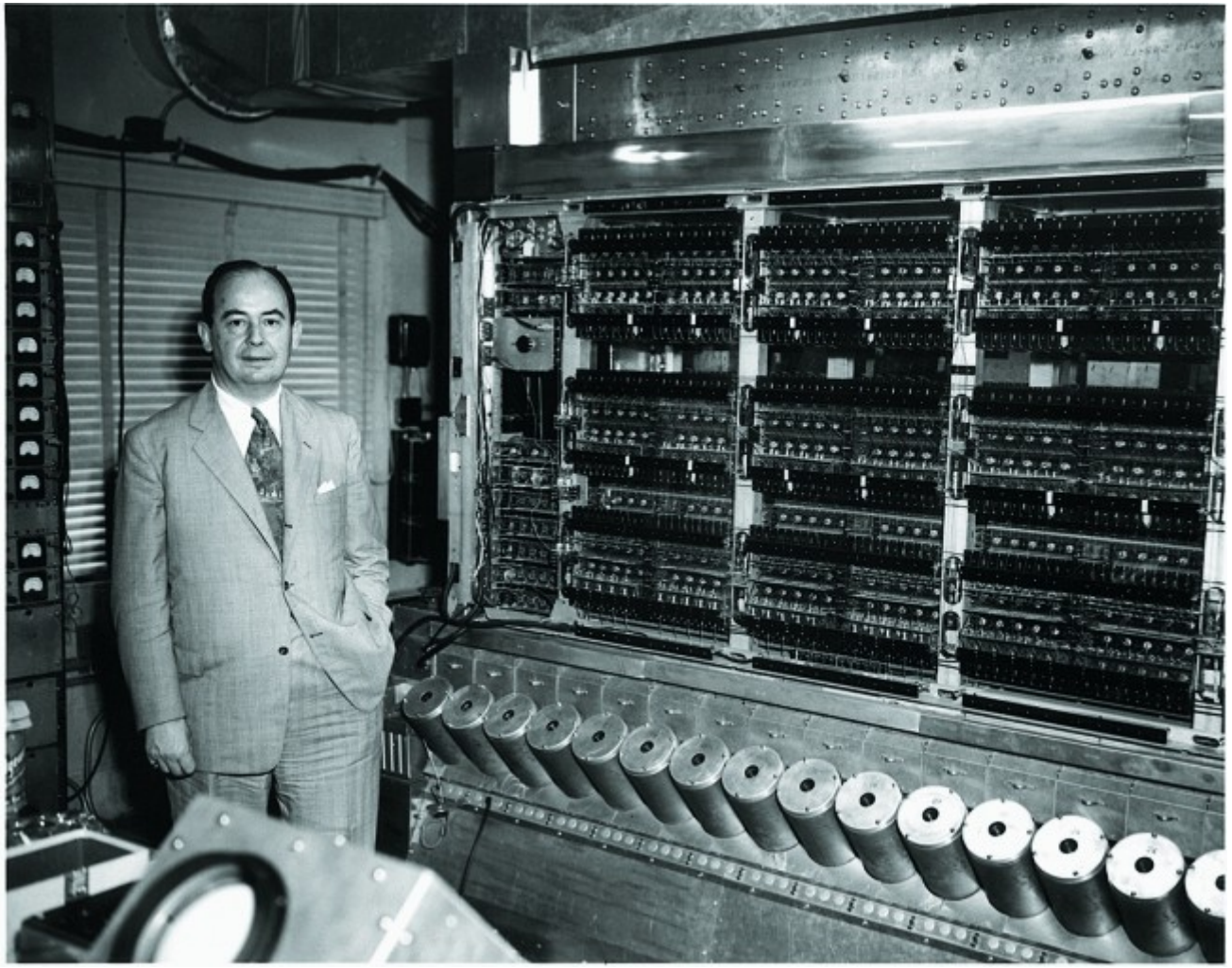
Merge sort



Prehľadávanie do hĺbky

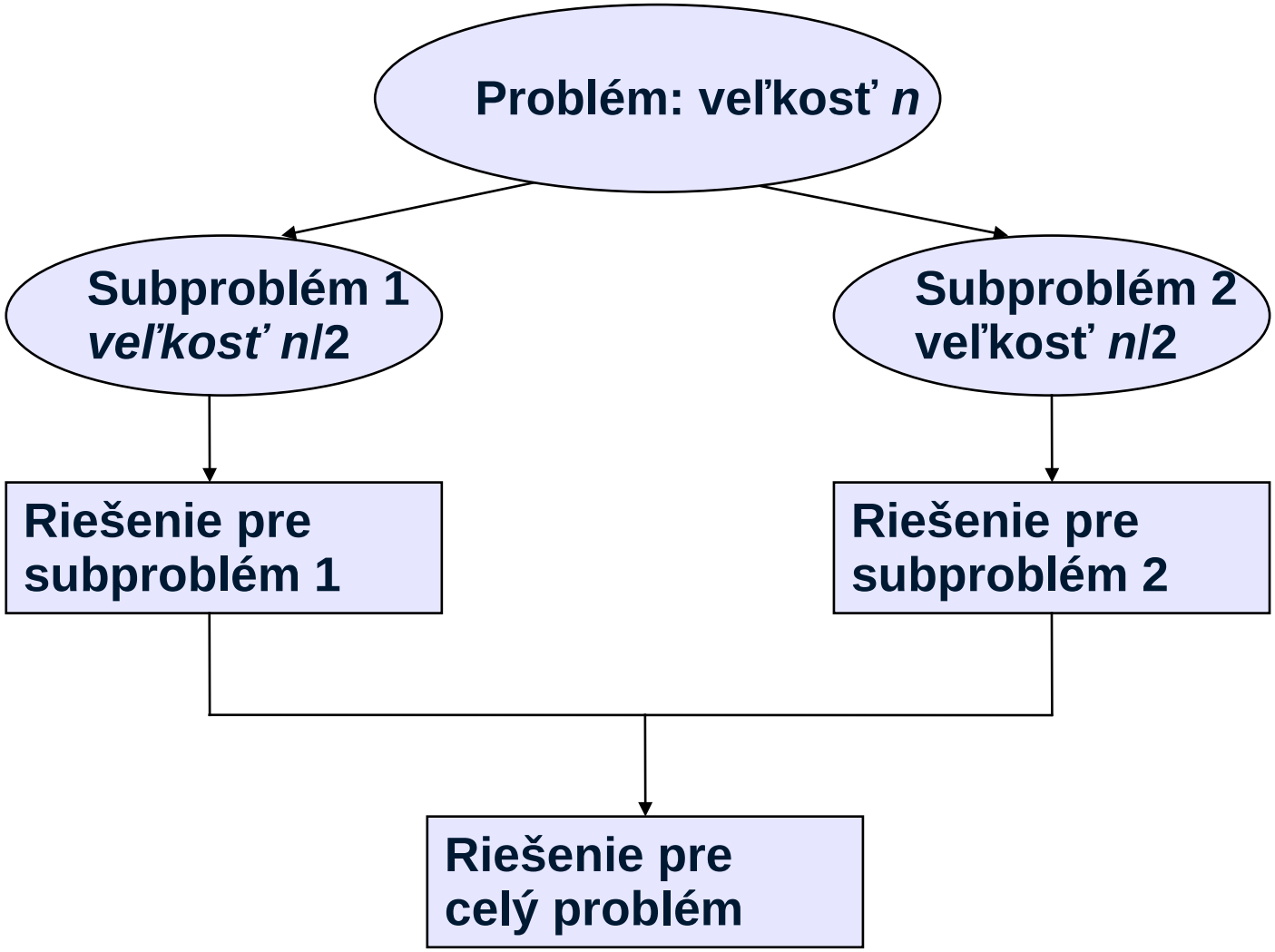
Rekurentná implementácia roztriedi pravý podstrom a potom pravý podstrom.

John von Neumann



Neumann János: merge sort 1945

Rozdeľuj a panuj



Quick sort

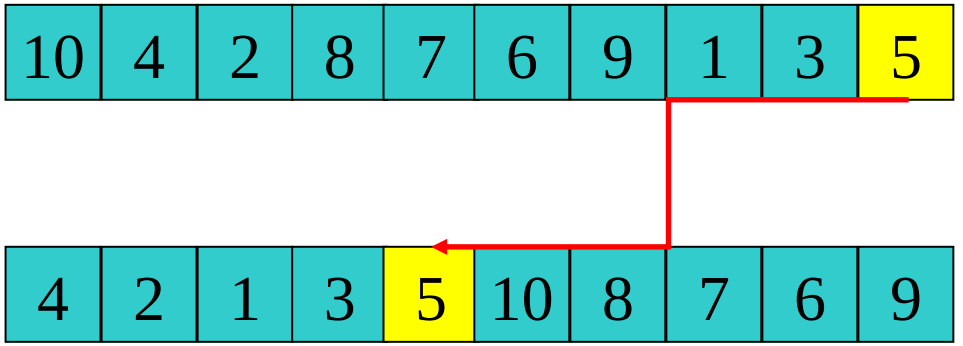
Idea:

Vyberiem z poľa prvok (pivot). Od pivota menšie prvky umiestnim naľavo a väčšie (rovné) napravo.

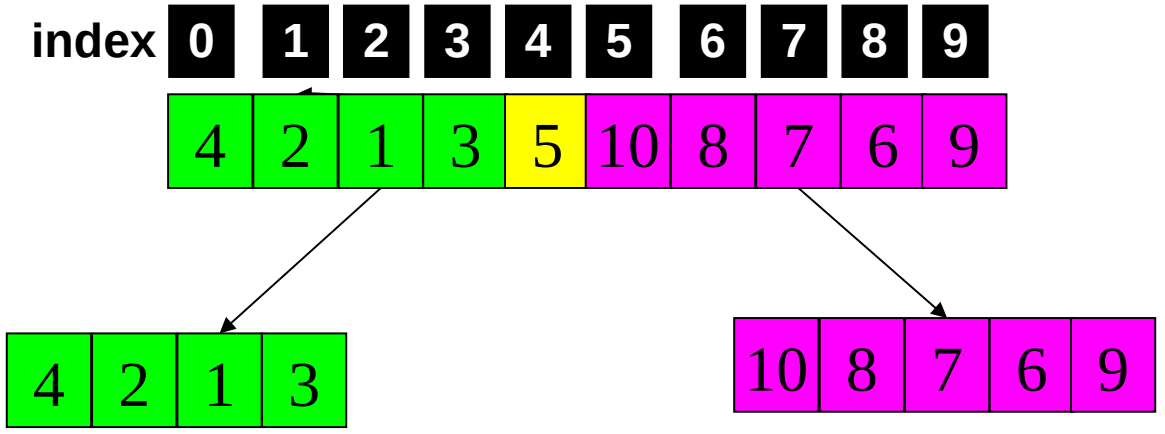
Aplikujem rekurzívne quick_sort zvlášť na menšie prvky a väčšie prvky.

Quick sort

1. quick_sort(0,9)



2. quick_sort(0,3)
   quick_sort(5,9)



Quick sort

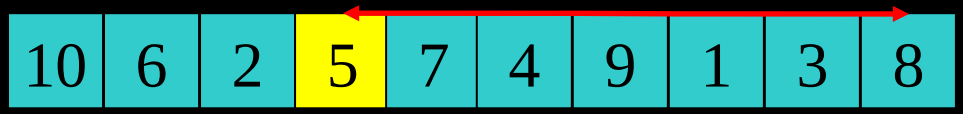
1. Vyberiem náhodne pivota a umiestnim ho na koniec poľa.

2. Prechod celým poľom a ukladanie za sebou na začiatok prvky menšie ako pivot.

3. Umiestnie pivota tesne za menšie prvky.

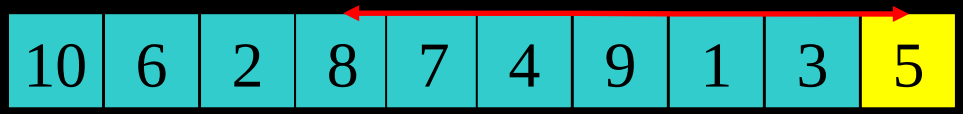
Quick sort

1. Vyberiem náhodne pivota a dám ho na koniec.



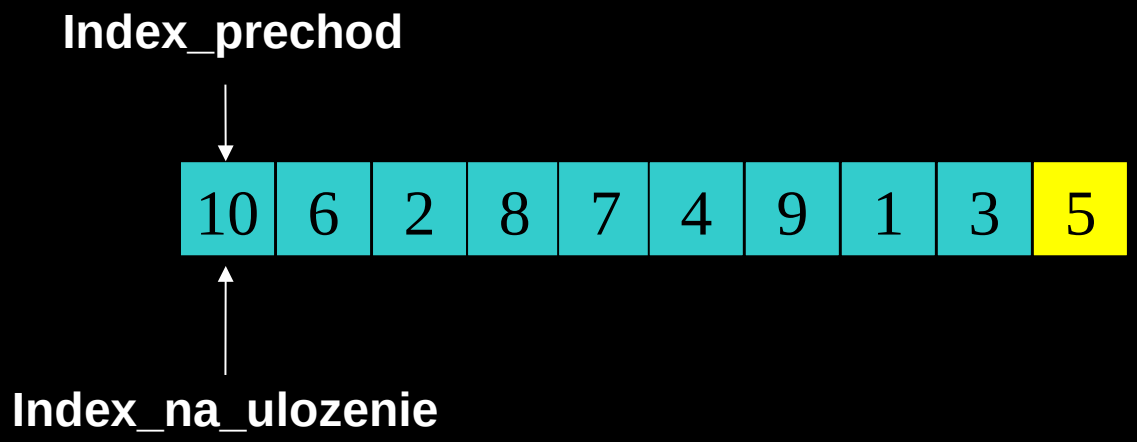
Quick sort

1. Vyberiem náhodne pivota a dám ho na koniec.

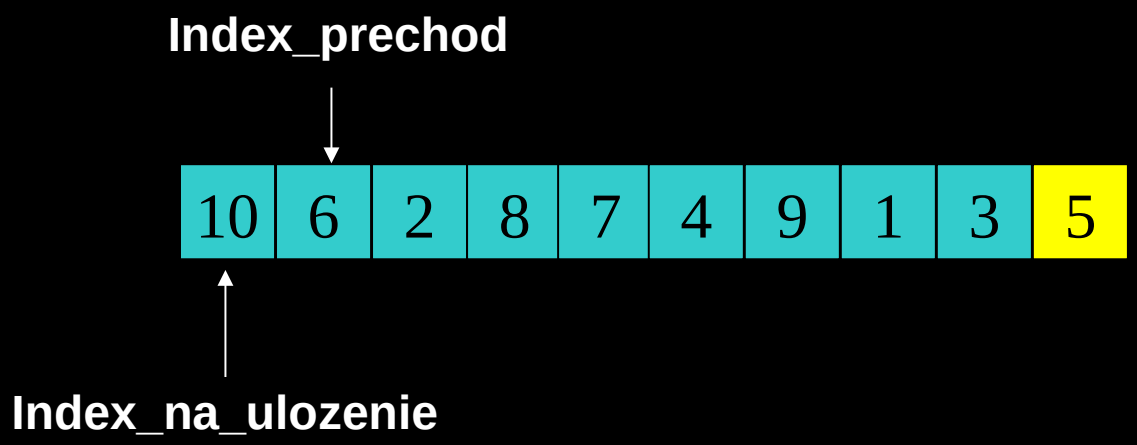


Quick sort

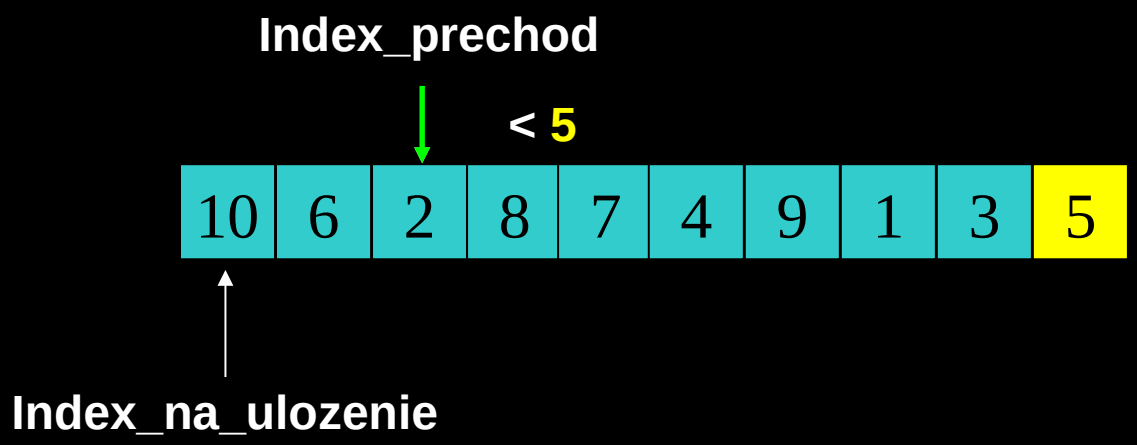
2. Prechod celým poľom a ukladanie za sebou na začiatok prvky menšie ako pivot.



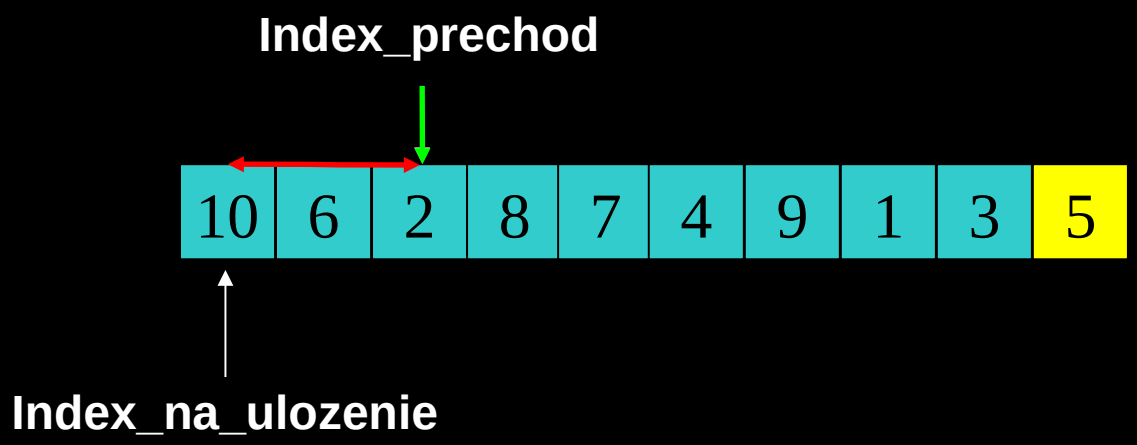
Quick sort



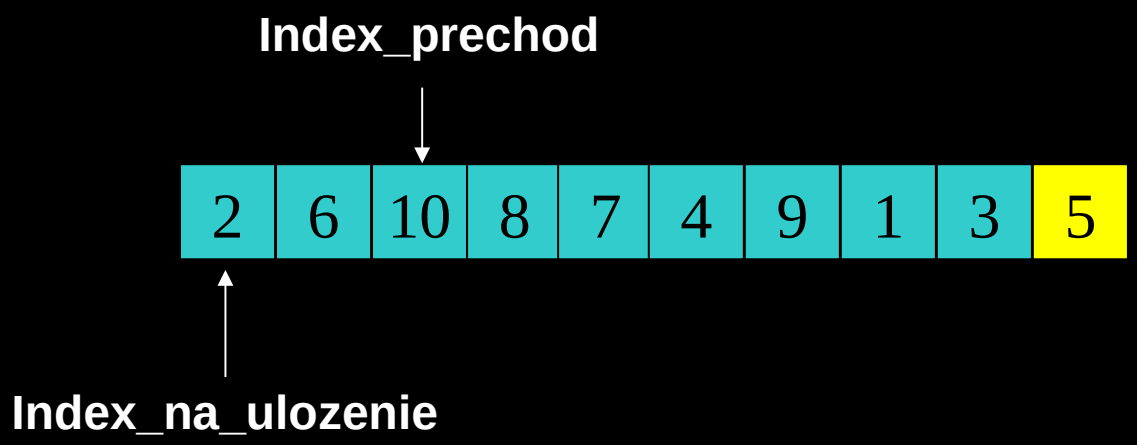
Quick sort



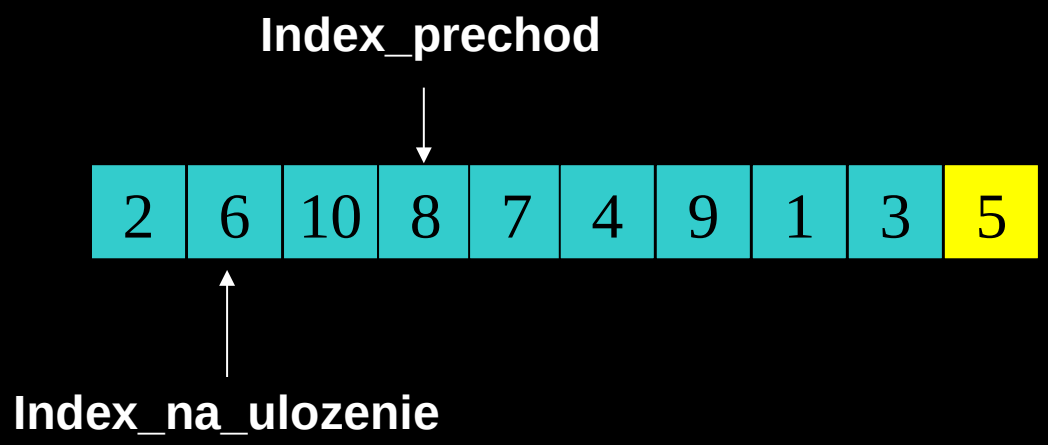
Quick sort



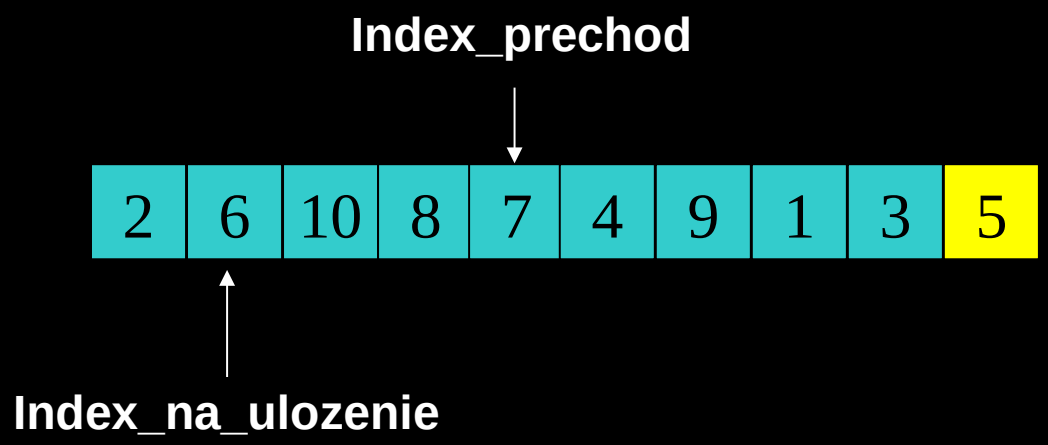
Quick sort



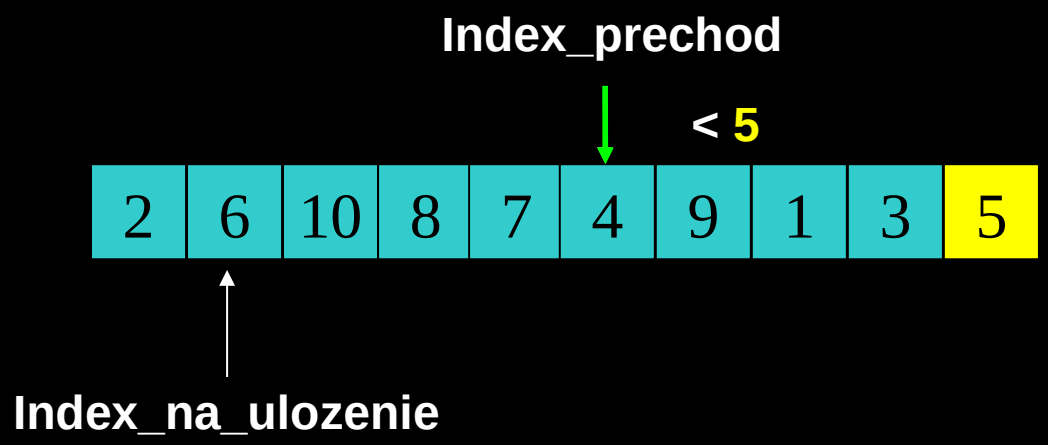
Quick sort



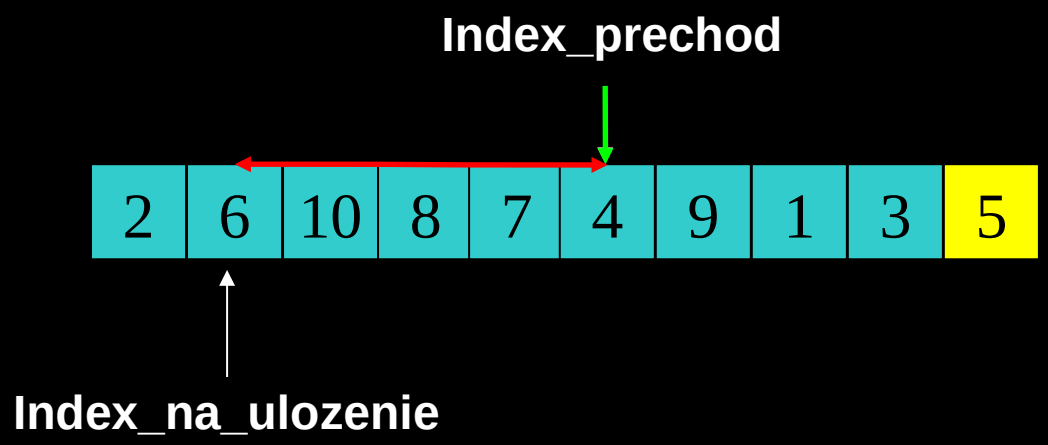
Quick sort



Quick sort



Quick sort



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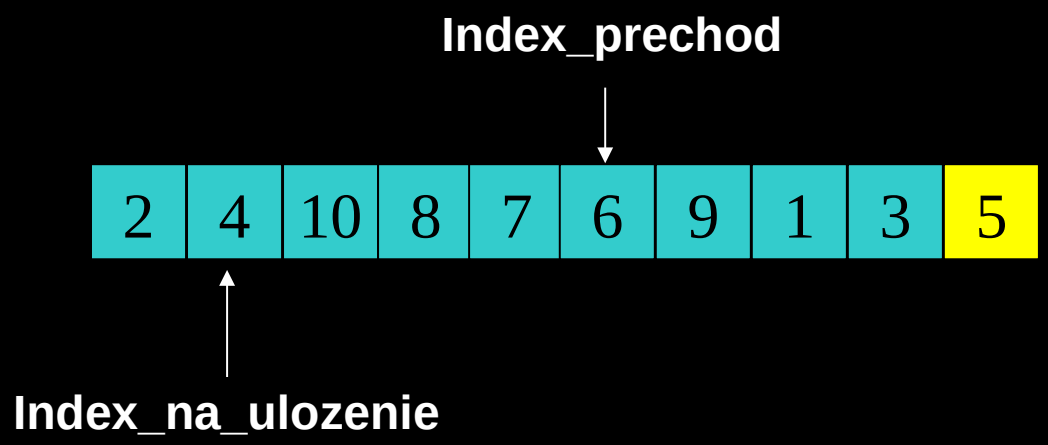

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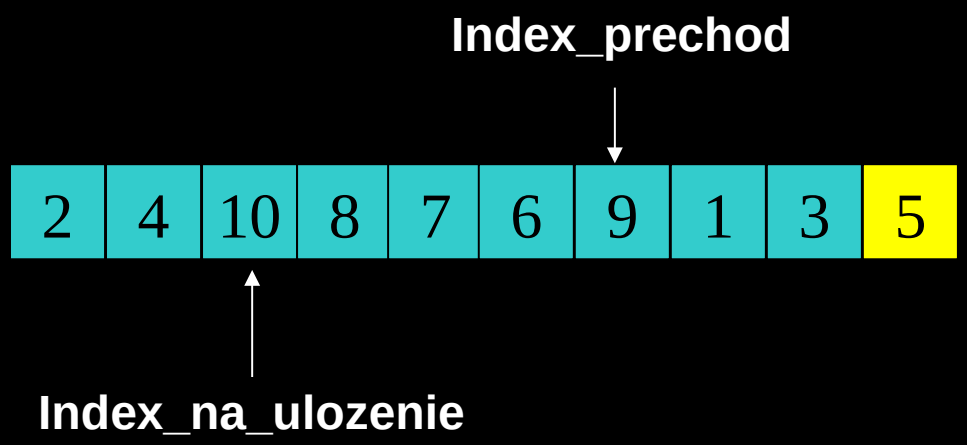

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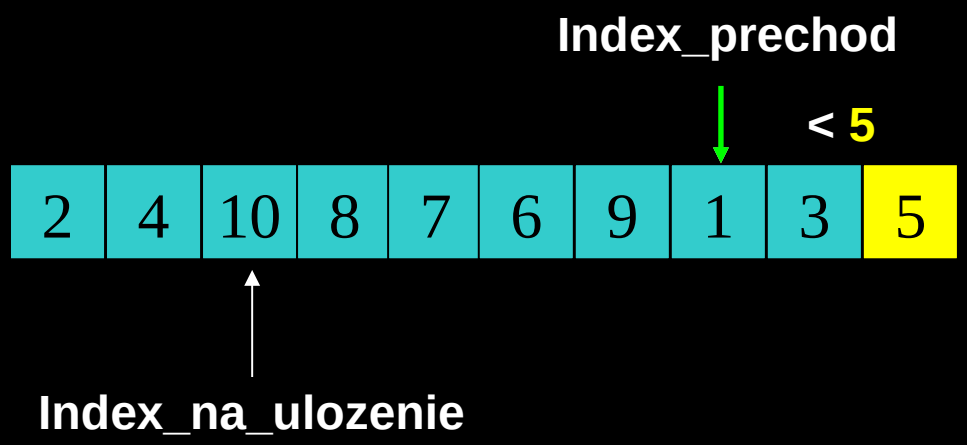

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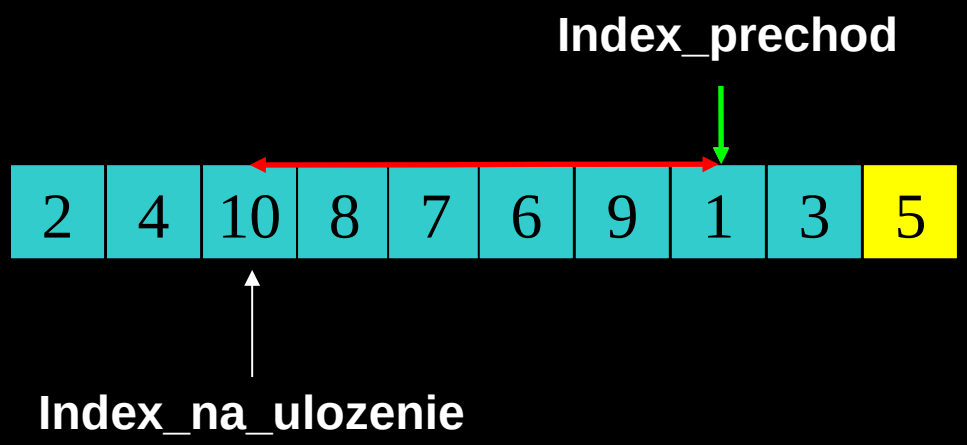
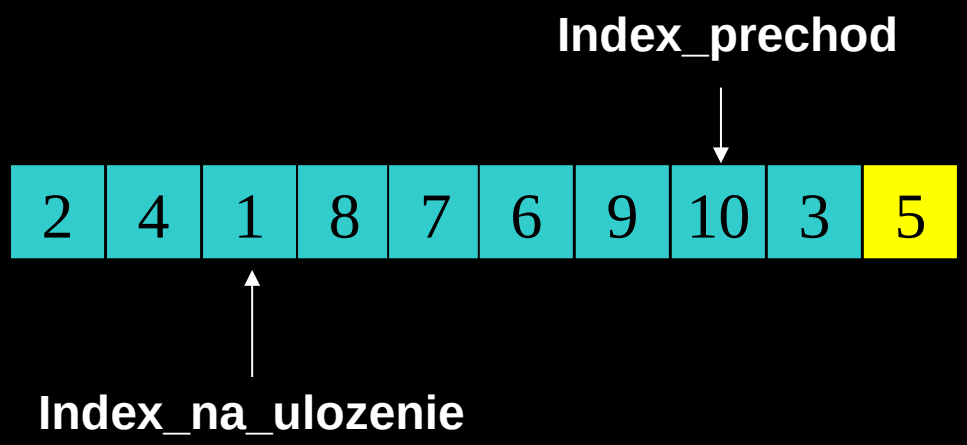


Quick sort



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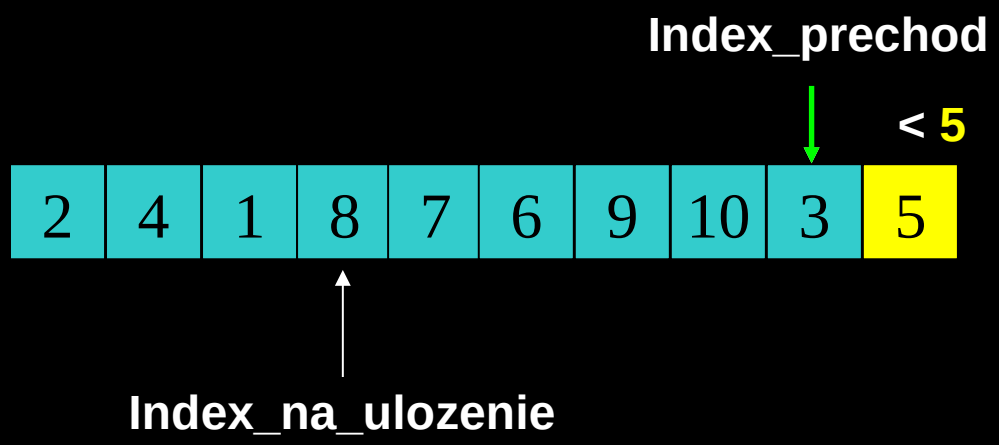
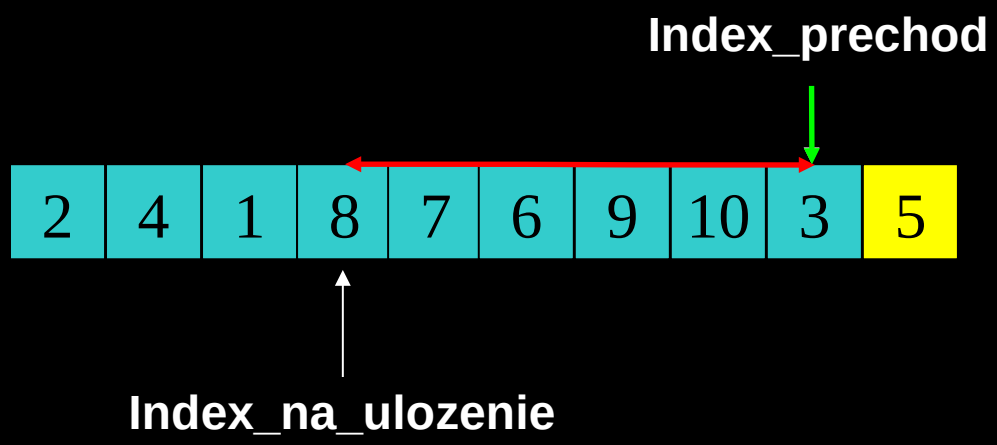
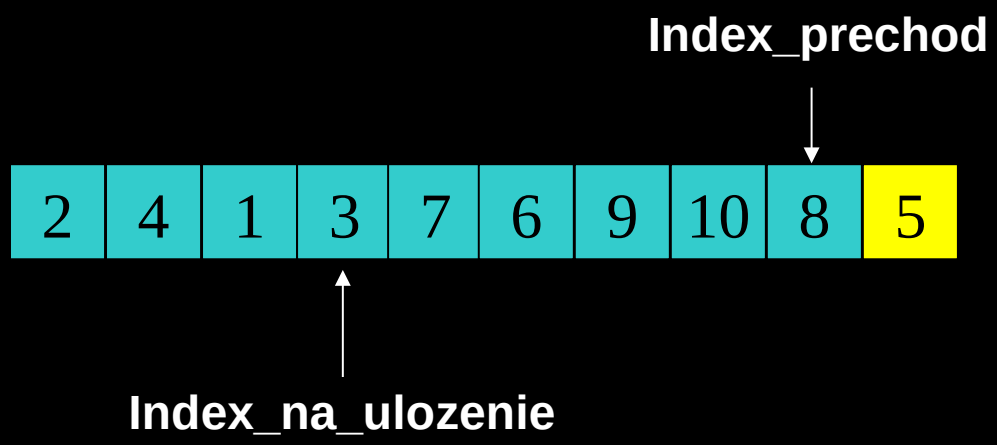


Quick sort



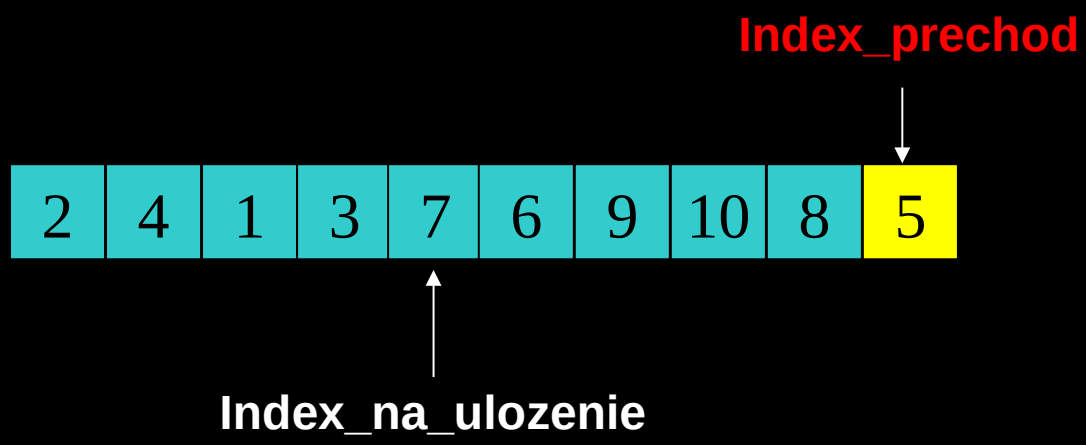
Quick sort



Quick sort

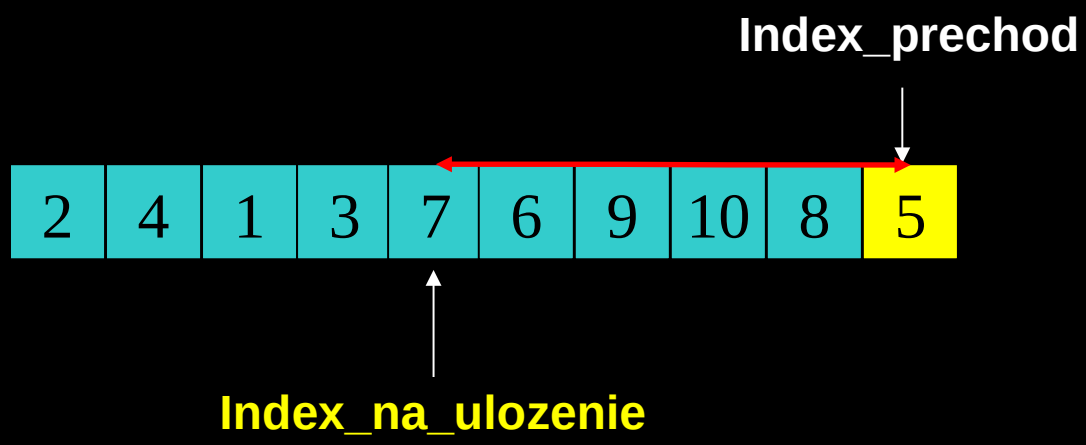
Koniec prechodu:

index_prechod = index pivota



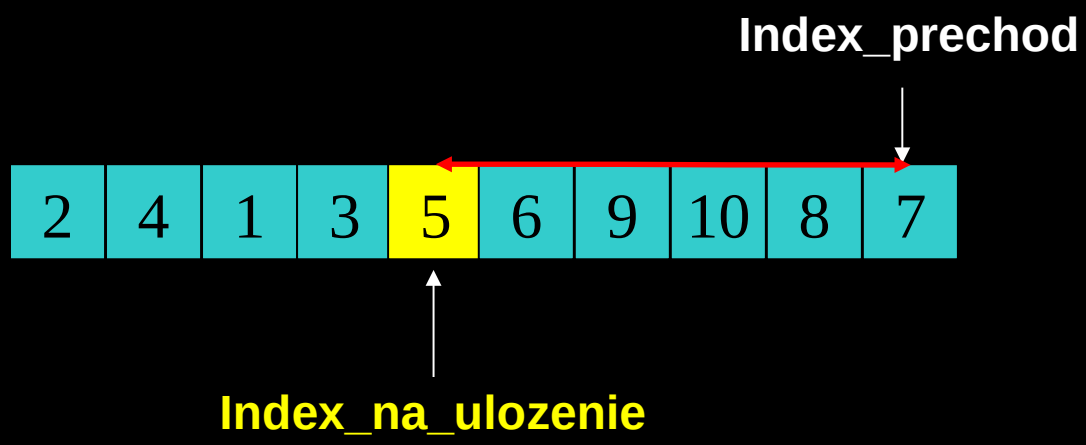
Quick sort

3. Umiestnim pivota na správne miesto index_na_umiestnenie



Quick sort

3. Umiestnim pivota na správne miesto index_na_umiestnenie



Voľba pivota

Prvok vľavo/vpravo: zlá voľba, ak je pole už usporiadané

Náhodná voľba

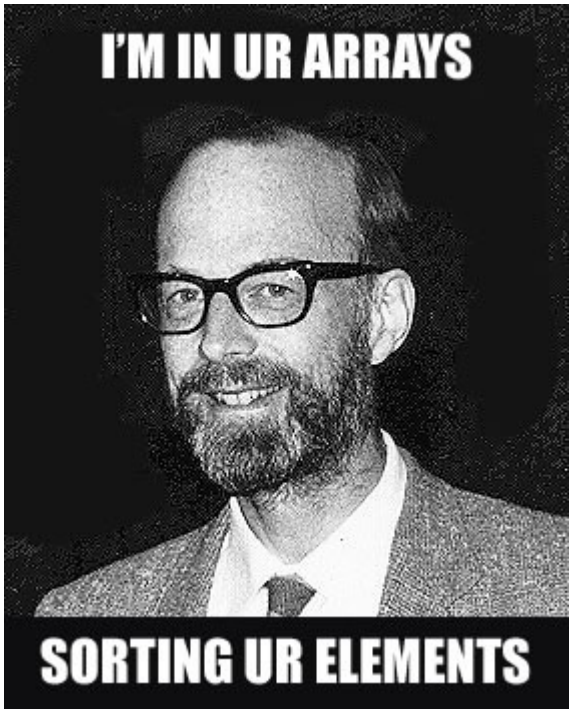
Medián: prvý, posledný a stredný prvok

Quick sort: zložitosť

Najhorší prípad: pivot je najväčší prvok v každom kroku, dve časti o veľkosti n-1 a 0: $O(n^2)$

Najlepší prípad: rozdelenie v každom kroku na dve rovnaké časti: $O(n \log n)$, hĺbka stromu volaní log n

Tony Hoare



Quick sort: 1960 počas študijného pobytu v ZSSR

Timeline

1945: merge sort (von Neumann)
1957: Fortran (John Backus)
1960: quick sort (Hoare)
1972: C (Dennis Ritchie)
1983: C++ (Stroustrup)
1995: Java (Gosling)