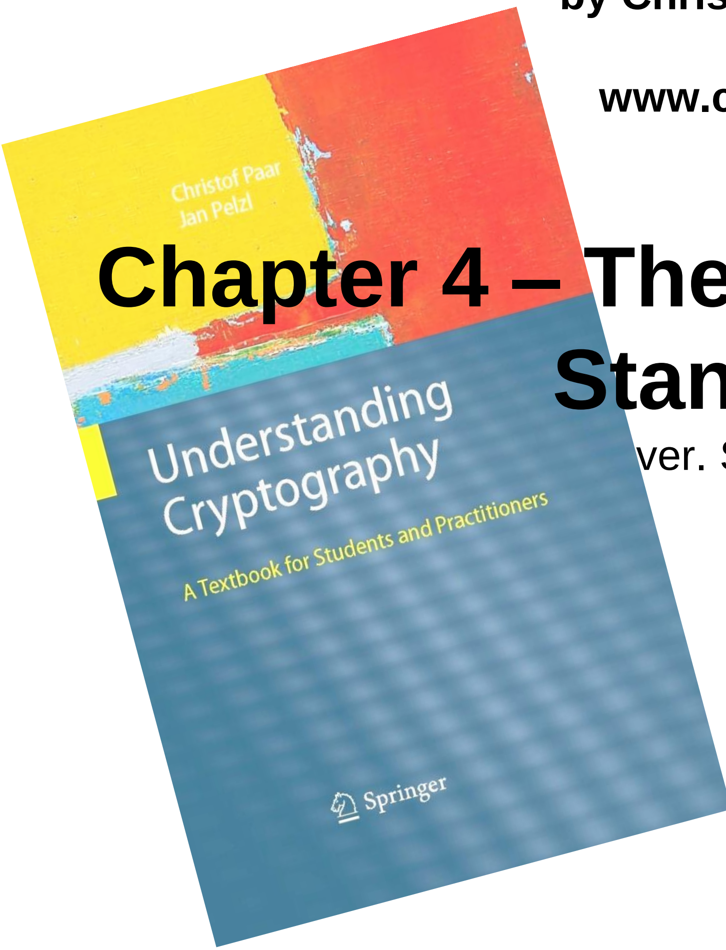
Understanding Cryptography

by Christof Paar and Jan Pelzl

www.crypto-textbook.com



Chapter 4 – The Advanced Encryption Standard (AES)

ver. September 27, 2024

These slides were originally prepared by Daehyun Strobel, Christof Paar and Jan Pelzl. Later, they were modified by Tomas Fabsic for purposes of teaching I-ZKRY at FEI STU.

Homework

- Read Chapter 4 (as well as Section 3.1).
- Solve problems from the exercise set no. 3 and submit them to AIS by **7.10.2024 23:59**.

Some legal stuff (sorry): Terms of Use

Content of this Chapter

- Overview of the AES algorithm
- Internal structure of AES
  - Byte Substitution layer
  - Diffusion layer
  - Key Addition layer
  - Key schedule
- Decryption

Content of this Chapter

- **Overview of the AES algorithm**
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Some Basic Facts

- AES is the most widely used symmetric cipher today

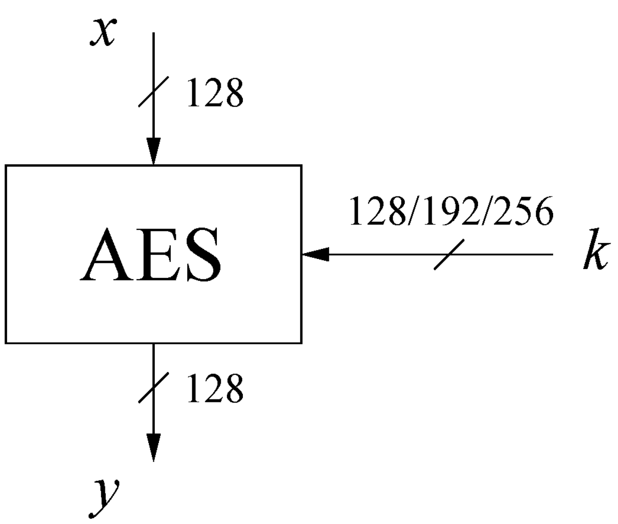
- The algorithm for AES was chosen by the US *National Institute of Standards and Technology* (NIST) in a multi-year selection process

- The requirements for all AES candidate submissions were:
  - Block cipher with **128-bit block size**
  - **Three supported key lengths**: 128, 192 and 256 bit
  - Security relative to other submitted algorithms
  - **Efficiency** in software and hardware

Chronology of the AES Selection

- The need for a new block cipher announced by NIST in January, 1997
- 15 candidates algorithms accepted in August, 1998
- 5 finalists announced in August, 1999:
  - *Mars* – IBM Corporation
  - *RC6* – *RSA* Laboratories
  - *Rijndael* – J. Daemen & V. Rijmen
  - *Serpent* – Eli Biham et al.
  - *Twofish* – B. Schneier et al.
- In October 2000, *Rijndael* was chosen as the AES
- AES was formally approved as a US federal standard in November 2001

AES: Overview

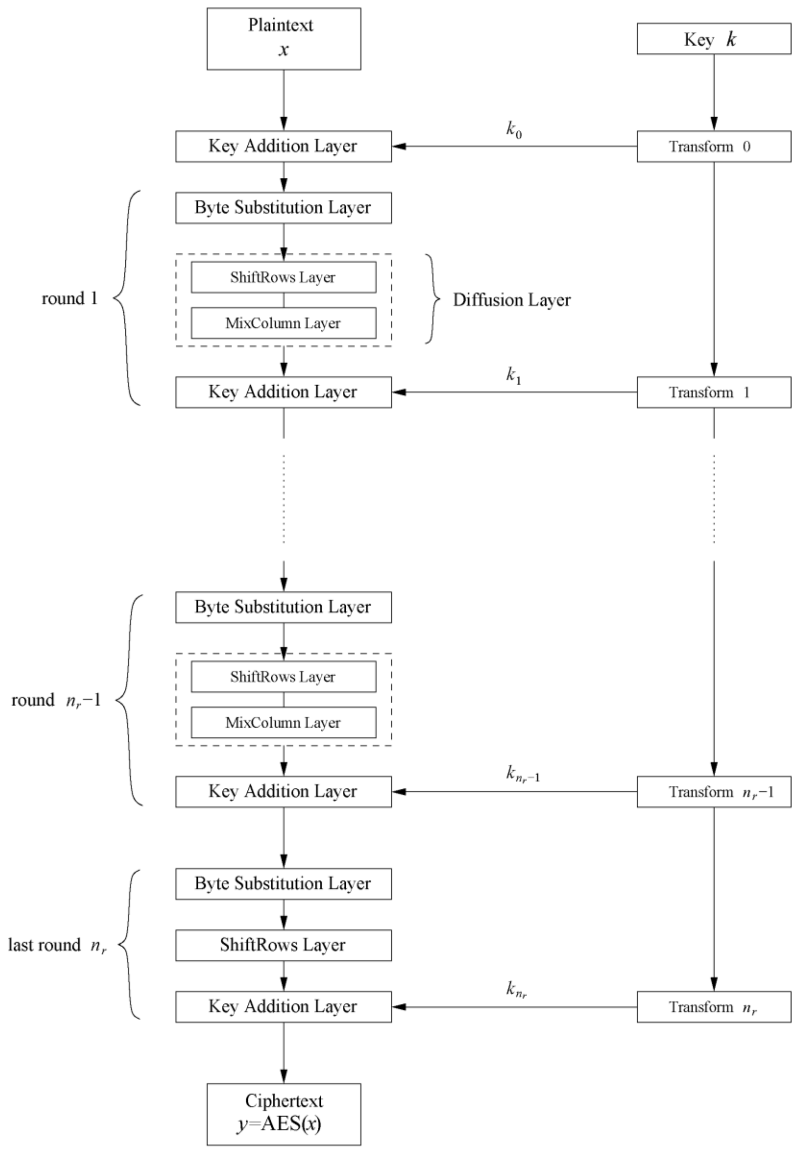


The number of rounds depends on the chosen key length:

Key length (bits)	Number of rounds
128	10
192	12
256	14

AES: Overview

- AES has a structure of a **substitution-permutation network (substitučno-permutačná sieť).**
- AES is an iterated cipher with 10/12/14 rounds
- Each round consists of “Layers”.
- At the end of each round a subkey (podkľúč) is used.
- All subkeys are derived from the original key *k.*
- A subkey is also added at before the 1st round and at the end of the last round (*key whitening*).



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- **Internal structure of AES**
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Internal Structure of AES

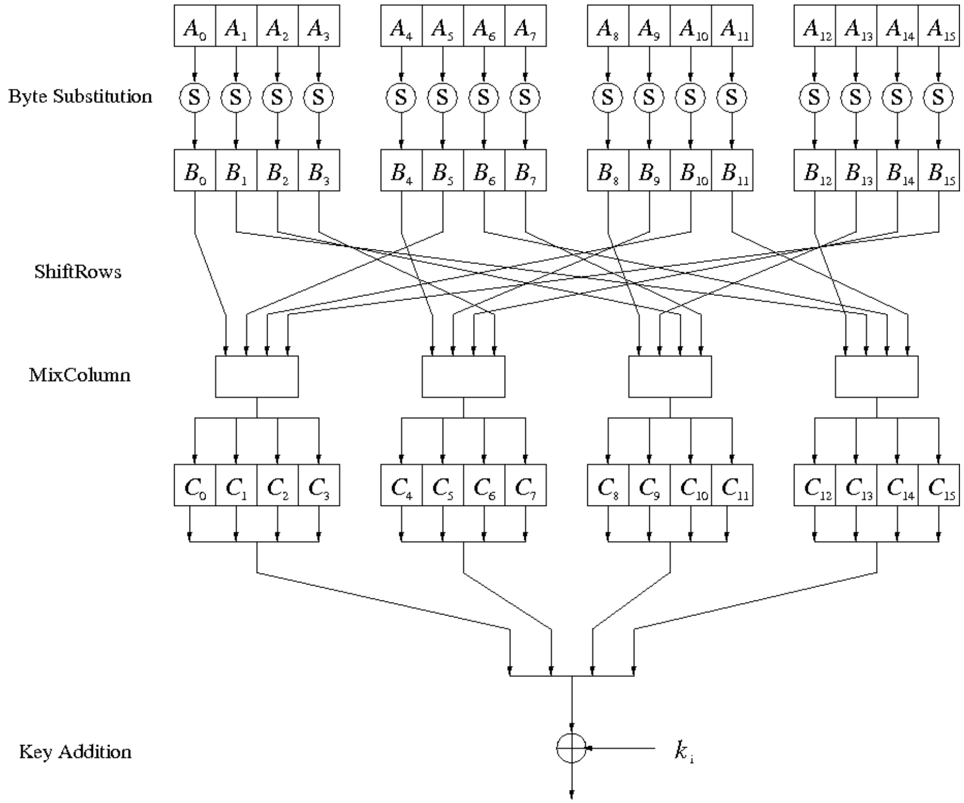
- AES is a byte-oriented cipher
- AES treats bytes as elements of GF(2^8)
- The state (stav) A (i.e., the 128-bit data path) can be arranged in a 4x4 matrix:

A_0	A_4	A_8	A_{12}
A_1	A_5	A_9	A_{13}
A_2	A_6	A_{10}	A_{14}
A_3	A_7	A_{11}	A_{15}

with $A_0,\ldots, A_{15}$ denoting the 16-byte input of AES

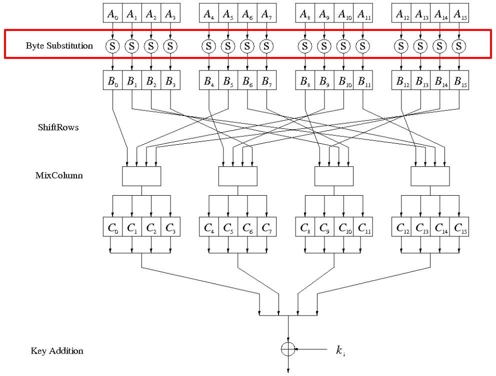
Internal Structure of AES

- Round function for rounds 1,2,…,n_{r-1}:



- Note: In the last round, the MixColumn tansformation is omitted

Byte Substitution Layer



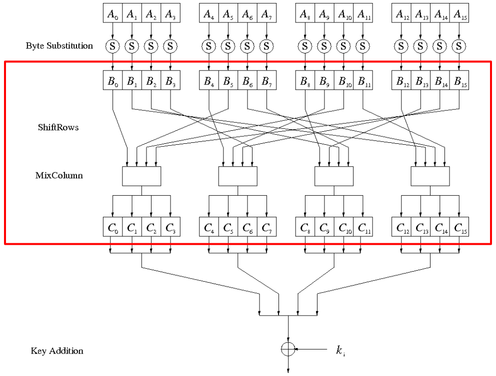
- The Byte Substitution layer consists of 16 **S-Boxes** with the following properties:

  The S-Boxes are

  - **identical**
  - the only **nonlinear** elements of AES, i.e., ByteSub(A_i) + ByteSub(A_j) ≠ ByteSub(A_i + A_j), for i,j = 0,…,15
  - **bijective**, i.e., there exists a one-to-one mapping of input and output bytes

    ⇒ S-Box can be uniquely reversed

Diffusion Layer

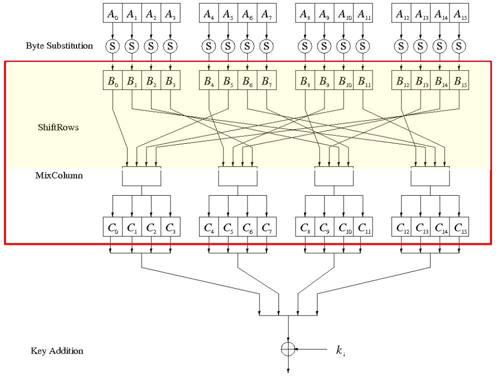


The Diffusion layer

- provides diffusion over all input state bits

- consists of two sublayers:
  - **ShiftRows Sublayer**: Permutation of the data on a byte level
  - **MixColumn Sublayer**: Matrix operation which combines (“mixes”) blocks of four bytes

ShiftRows Sublayer



- Rows of the state matrix are shifted cyclically:

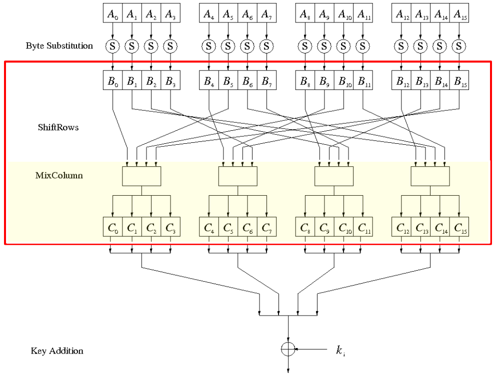
Input matrix

B_0	B_4	B_8	B_{12}
B_1	B_5	B_9	B_{13}
B_2	B_6	B_{10}	B_{14}
B_3	B_7	B_{11}	B_{15}

Output matrix

B_0	B_4	B_8	B_{12}	no shift
B_5	B_9	B_{13}	B_1	← one position left shift
B_{10}	B_{14}	B_2	B_6	← two positions left shift
B_{15}	B_3	B_7	B_{11}	← three positions left shift

MixColumn Sublayer



- Linear transformation which mixes each column of the state matrix

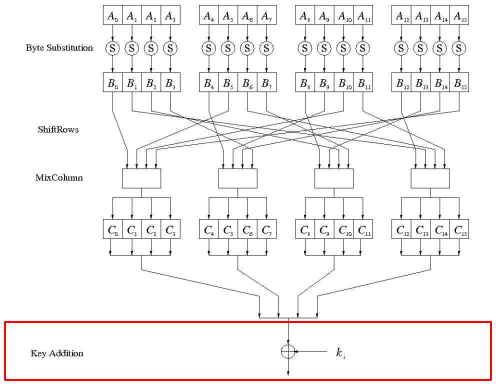
- Each 4-byte column is considered as a vector and multiplied by a fixed 4x4 matrix, e.g.,

$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \cdot \begin{pmatrix} B_0 \\ B_5 \\ B_{10} \\ B_{15} \end{pmatrix}$$

where 01, 02 and 03 are given in hexadecimal notation

- All arithmetic is done in the Galois field $GF(2^8)$ (for more information see Chapter 4.3 in *Understanding Cryptography*)

Key Addition Layer



- Inputs:
  - 16-byte state matrix C
  - 16-byte subkey k_i
- Output: $C \oplus k_i$
- The subkeys are generated in the key schedule

Key Schedule

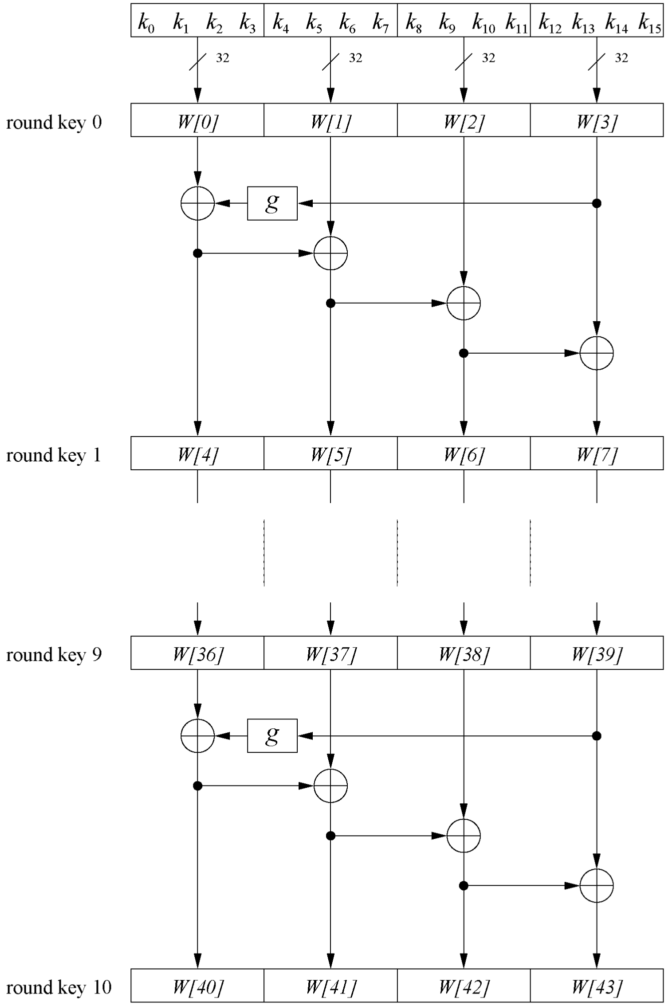
- Subkeys are derived recursively from the original 128/192/256-bit input key
- Each round has 1 subkey, plus 1 subkey at the beginning of AES

Key length (bits)	Number of subkeys
128	11
192	13
256	15

- Key whitening: Subkey is used both at the input and output of AES
  $\Rightarrow$ # subkeys = # rounds + 1
- There are different key schedules for the different key sizes

Key Schedule

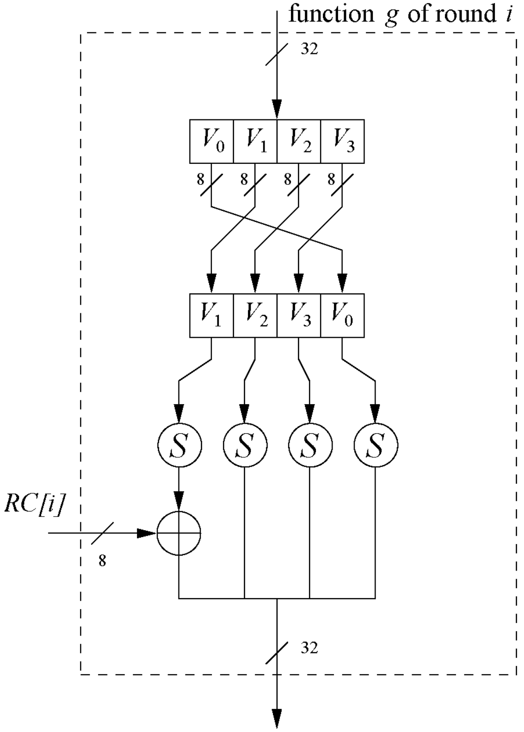
Example: Key schedule for 128-bit key AES



- Word-oriented: 1 word = 32 bits
- 11 subkeys are stored in *W[0]…W[3], W[4]…W[7], … , W[40]…W[43]*
- First subkey *W[0]…W[3]* is the original AES key

Key Schedule

- Function *g* rotates its four input bytes and performs a bytewise S-Box substitution $\Rightarrow$ nonlinearity



- The round coefficient *RC* is only added to the leftmost byte and varies from round to round:

$$RC[1] = x^0 = (00000001)_2$$
$$RC[2] = x^1 = (00000010)_2$$
$$RC[3] = x^2 = (00000100)_2$$
...
$$RC[10] = x^9 = (00110110)_2$$

- x^i represents an element in a Galois field (again, cf. Chapter 4.3 of *Understanding Cryptography*)

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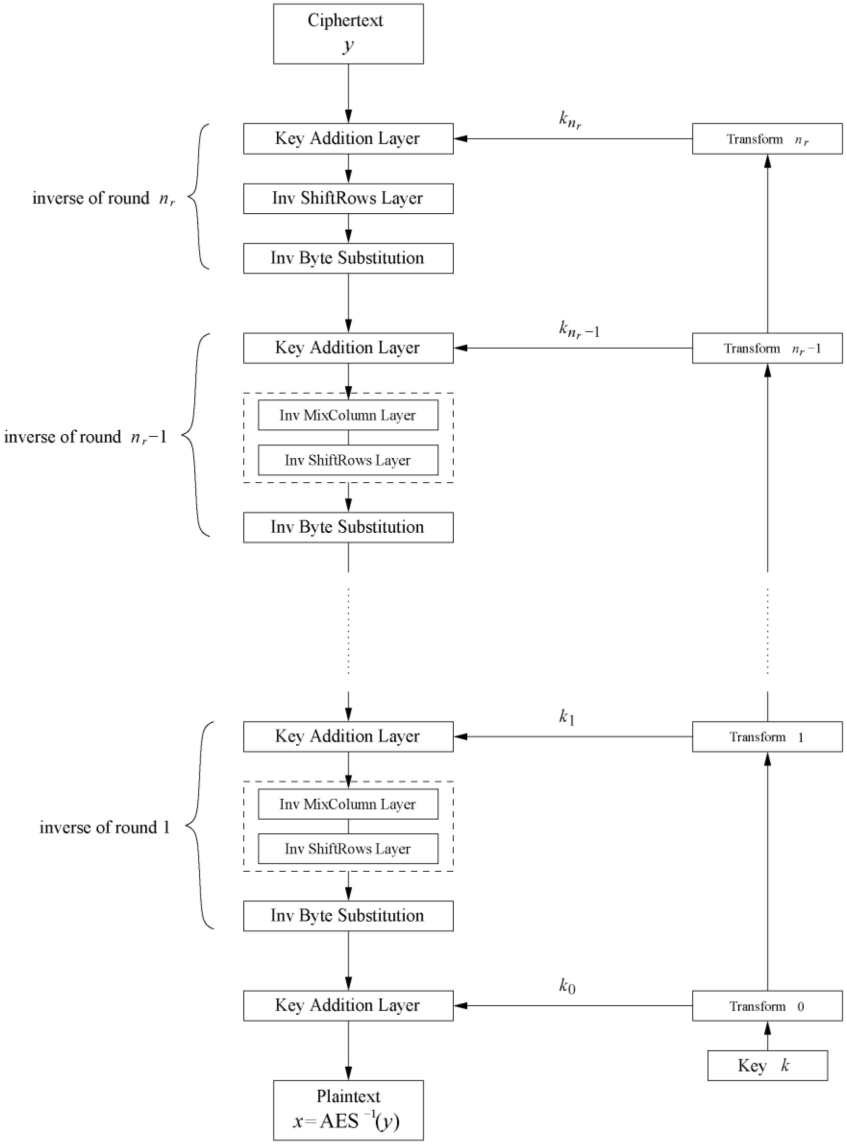
- **Decryption**

Decryption



- AES is not based on a Feistel network

⇒ All layers must be inverted for decryption:

- MixColumn layer → **Inv MixColumn layer**
- ShiftRows layer→ **Inv ShiftRows layer**
- Byte Substitution layer → **Inv Byte Substitution layer**
- Key Addition layer is its own inverse

Decryption

- **Inv MixColumn layer**:
  - To reverse the MixColumn operation, each column of the state matrix C must be multiplied with the **inverse of the 4x4 matrix**, e.g.,

$$\begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \cdot \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

where 09, 0*B, 0D* and 0*E* are given in hexadecimal notation

- Again, all arithmetic is done in the Galois field $GF(2^8)$ (for more information see Chapter 4.3 in *Understanding Cryptography*)

Decryption

- **Inv ShiftRows layer**:
  - All rows of the state matrix B are shifted to the opposite direction:

Input matrix

B_0	B_4	B_8	B_{12}
B_1	B_5	B_9	B_{13}
B_2	B_6	B_{10}	B_{14}
B_3	B_7	B_{11}	B_{15}

Output matrix

B_0	B_4	B_8	B_{12}	no shift
B_{13}	B_1	B_5	B_9	→ one position right shift
B_{10}	B_{14}	B_2	B_6	→ two positions right shift
B_7	B_{11}	B_{15}	B_3	→ three positions right shift

Decryption

- **Inv Byte Substitution layer**:
  - Since the S-Box is bijective, it is possible to construct an inverse, such that

    $$A_i = S^{-1}(B_i) = S^{-1}(S(A_i))$$

    $\Rightarrow$ The inverse S-Box is used for decryption. It is usually realized as a lookup table

- **Decryption key schedule**:
  - Subkeys are needed in reversed order (compared to encryption)
  - In practice, for encryption and decryption, the same key schedule is used. This requires that all subkeys must be computed before the encryption of the first block can begin

Lessons Learned

- AES is a modern block cipher which supports three key lengths of 128, 192 and 256 bit. It provides excellent long-term security against brute-force attacks.
- AES has been studied intensively since the late 1990s and no attacks have been found that are better than brute-force.
- AES is not based on Feistel networks. Its basic operations use Galois field arithmetic and provide strong diffusion and confusion.
- AES is part of numerous open standards such as IPsec or TLS, in addition to being the mandatory encryption algorithm for US government applications. It seems likely that the cipher will be the dominant encryption algorithm for many years to come.
- AES is efficient in software and hardware.