

# DUS

- diskkrétne
- udalostné
- systémy

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Faculty of Electrical Engineering and Information Technology

# Semantics of PN

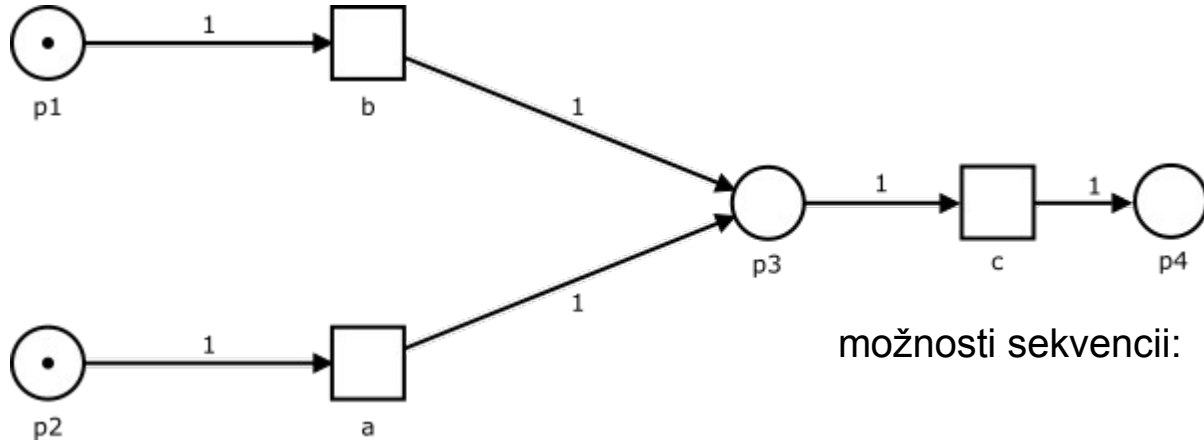
## Sekvenčná sémantika

- sekvencia spustení

## Nesekvenčná sémantika

- kroková sekvencia
- výrazy
- procesy
- označené čiastočné usporiadanie (multimnožiny prechodov)

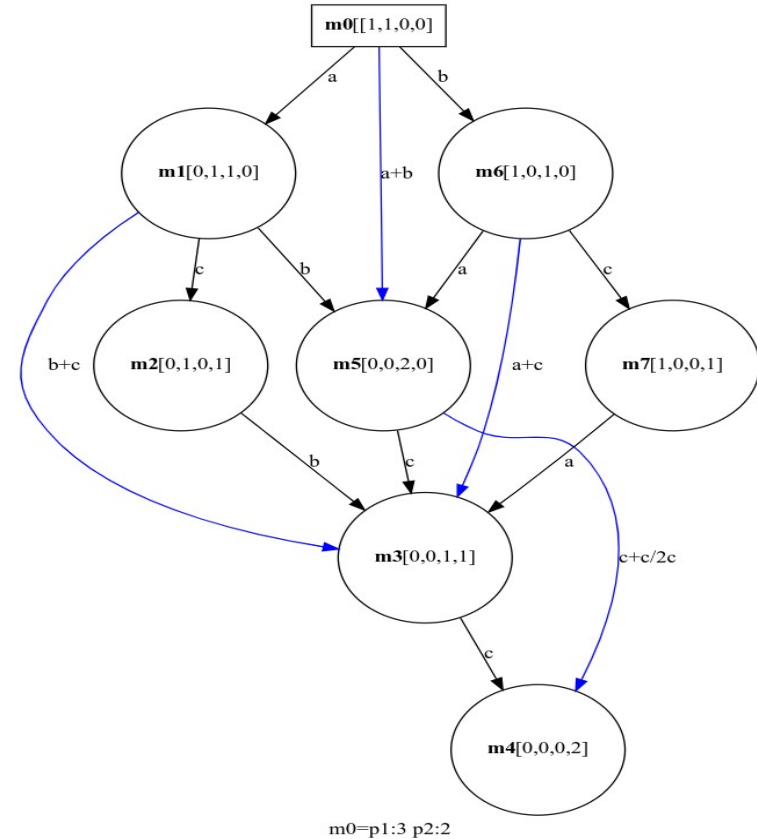
# Steps sequence semantics



možnosti sekvenci:

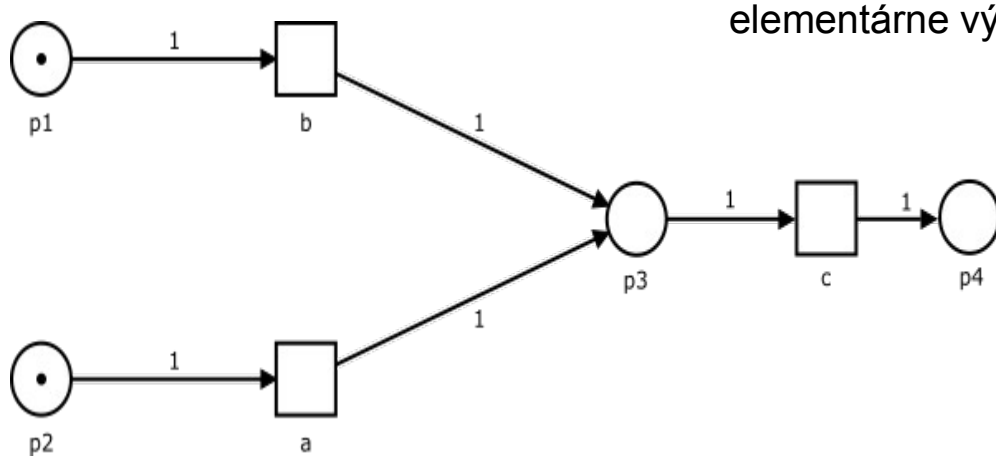
prepisovacie pravidlá:

- kauzalita ;
- paralelizmus +

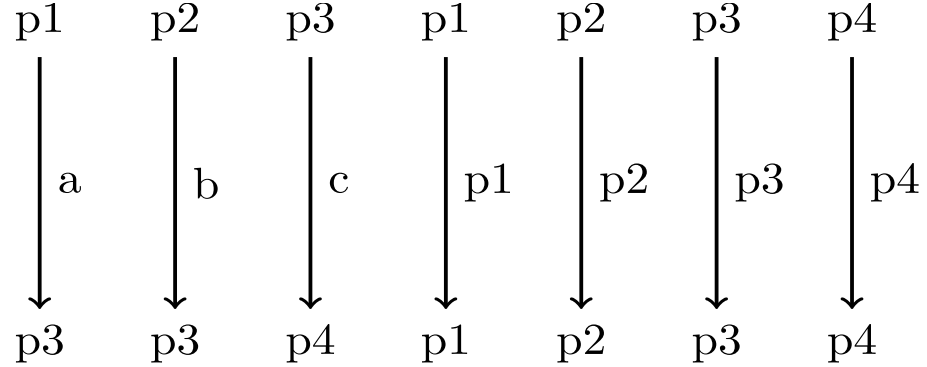


$m_0=p_1:3 \ p_2:2$

# Algebraic terms



elementárne výrazy:



$p_1 + p_2$



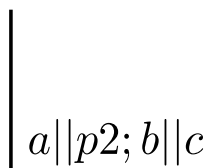
$p_2 + p_3$

$p_2 + p_3$



$p_3 + p_4$

$p_1 + p_2$



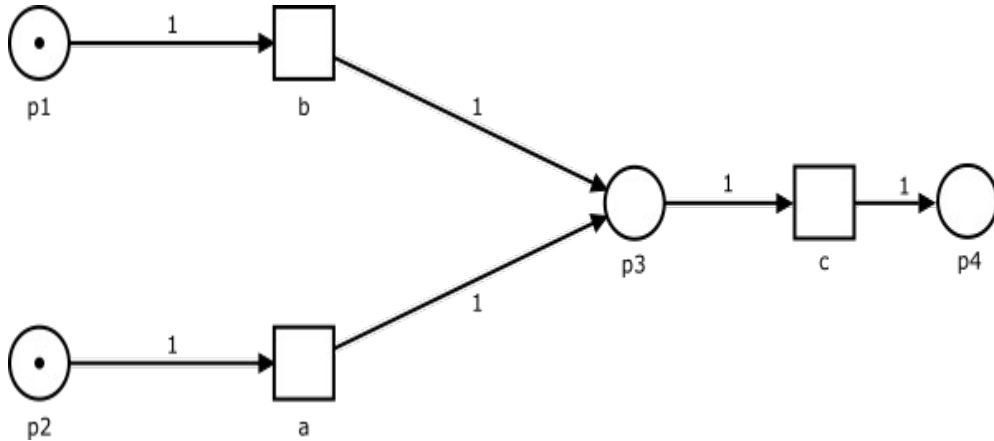
$p_3 + p_4$

$p_1 + p_2$



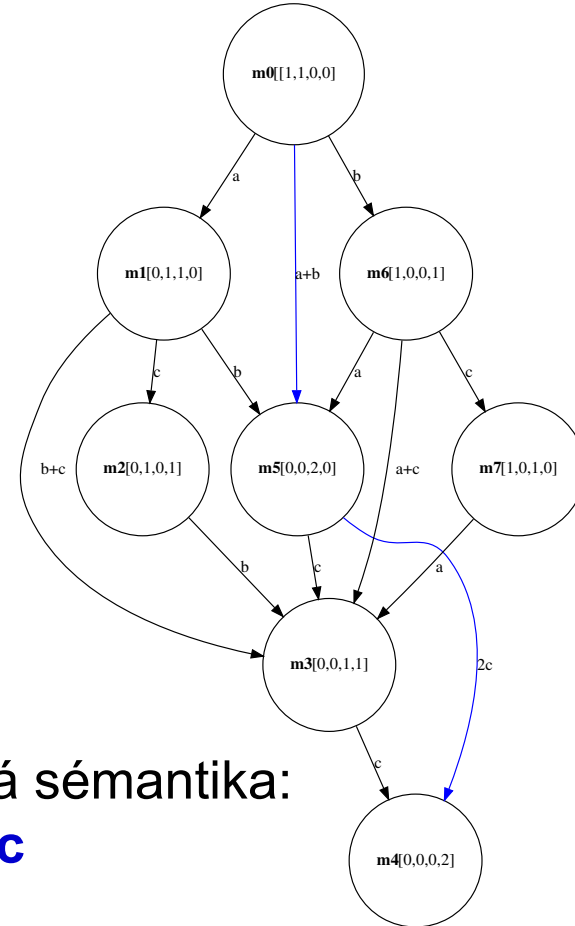
$2p_4$

# Semantics



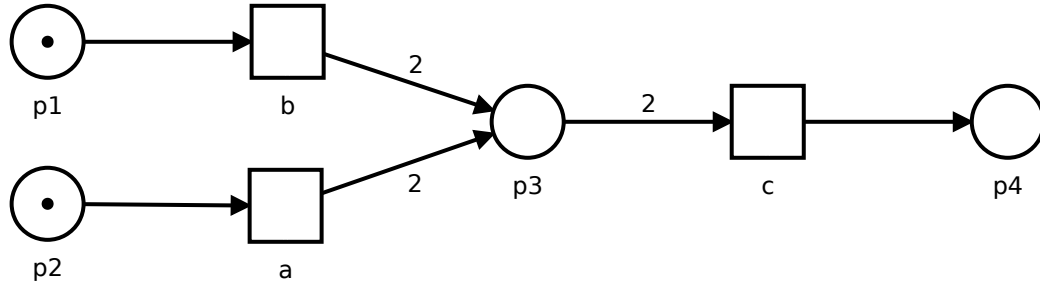
Výrazova sémantika:

$$p_1 + p_2 \xrightarrow{(a||b);(2c)} 2p_4$$



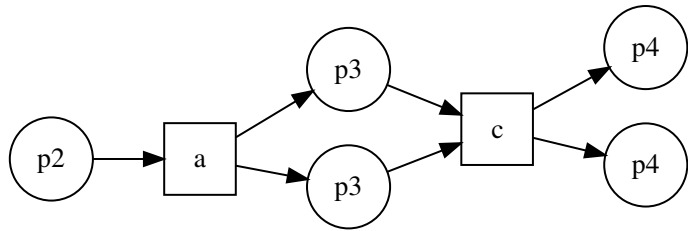
Kroková sémantika:  
 **$a+b; c+c$**

# Process semantic



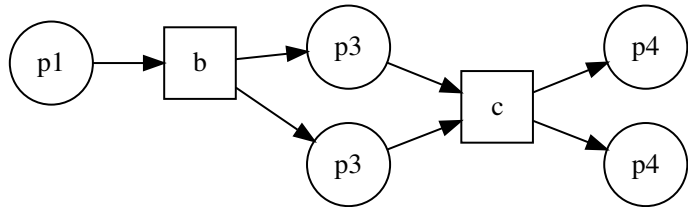
Výrazova sémantika:

$$p1 + p2 \xrightarrow{(a||b);(2c)} 2p4$$

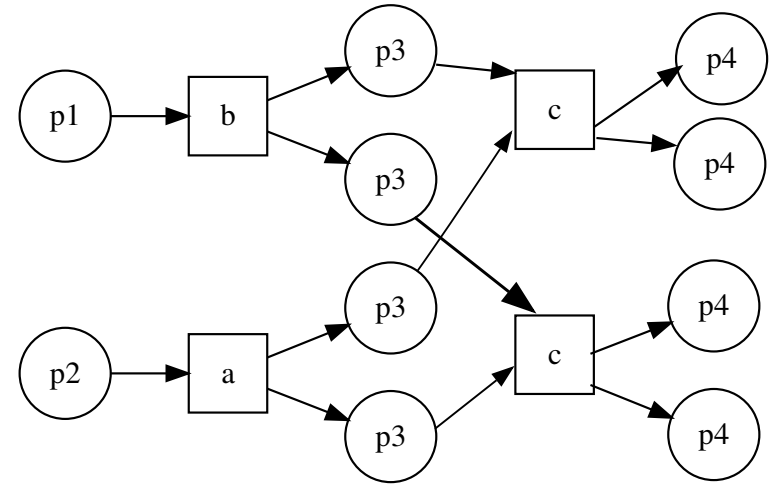


Procesná sémantika  
toho istého výrazu.

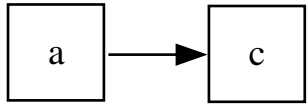
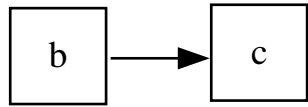
a,



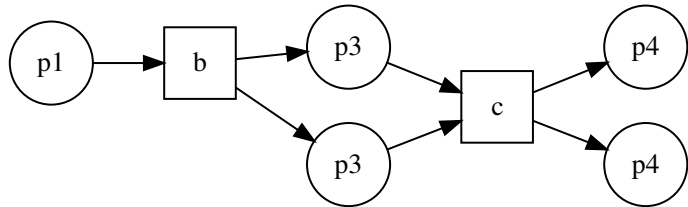
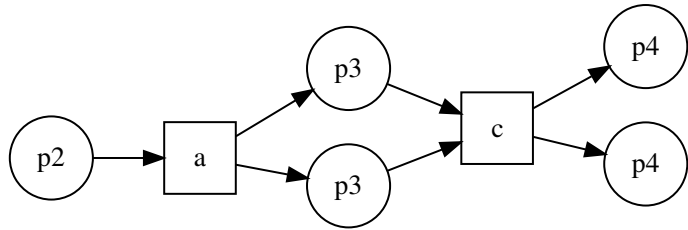
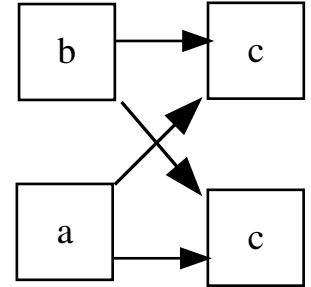
b,



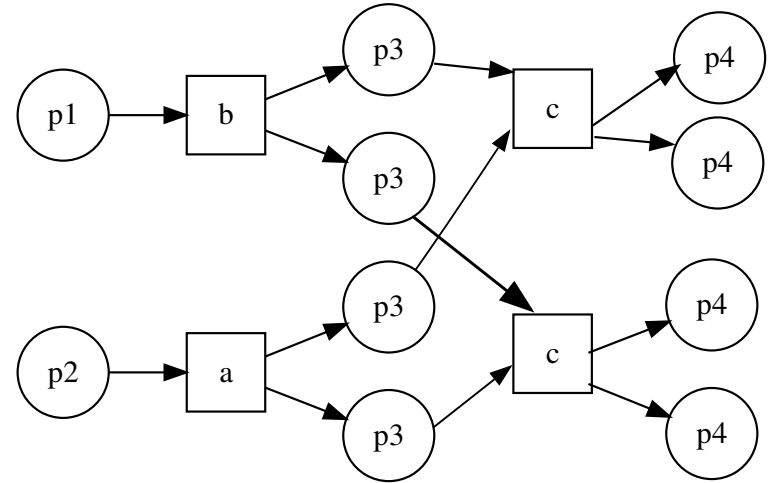
# Process semantic



behy



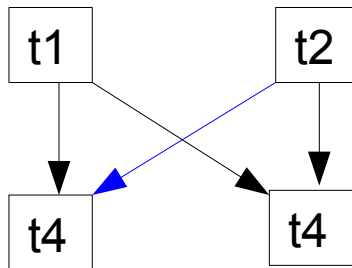
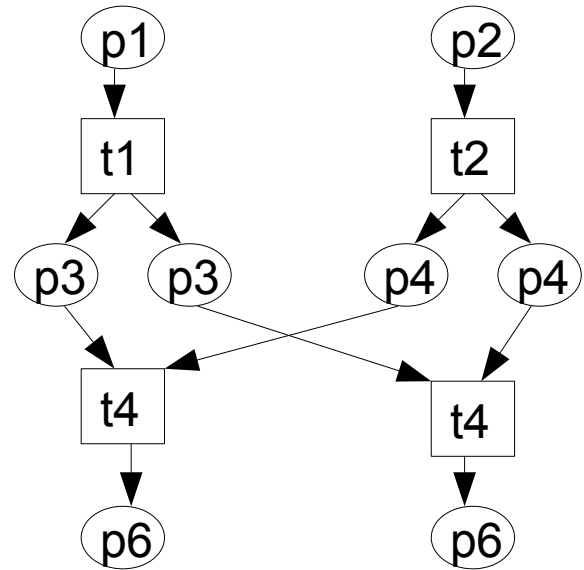
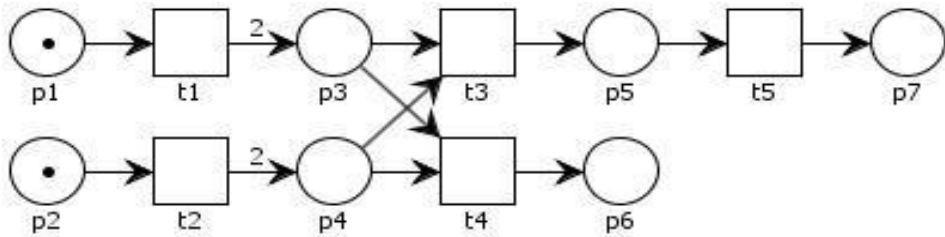
procesy



# Process semantics

Určite či je proces/beh spustiteľný v PS.

N-free LPO  $\rightarrow$  beh + značky v miestach = proces





# DAG / LPO

**Čiastočné usporiadanie** je dvojica  $(V, \rightarrow)$ , kde

$V$  je množina vrcholov/**udalostí**,

$\rightarrow$  je **binárna relácia** na  $V$  ( $\rightarrow$  je podmnožina  $V \times V$ , t.j. podmnožina množiny všetkých usporiadaných dvojíc prvkov z  $V$ , skutočnosť, že dvojica  $(v, v')$  je v relácii  $\rightarrow$  graficky znázorňujeme ako  $v \rightarrow v'$ ),

pričom relácia  $\rightarrow$  je

**ireflexívna**. ( $\forall v \in V: (v, v) \notin \rightarrow$ )

**tranzitívna**. ( $x, y, z \in V: x \rightarrow y \wedge y \rightarrow z \Rightarrow x \rightarrow z$ ).

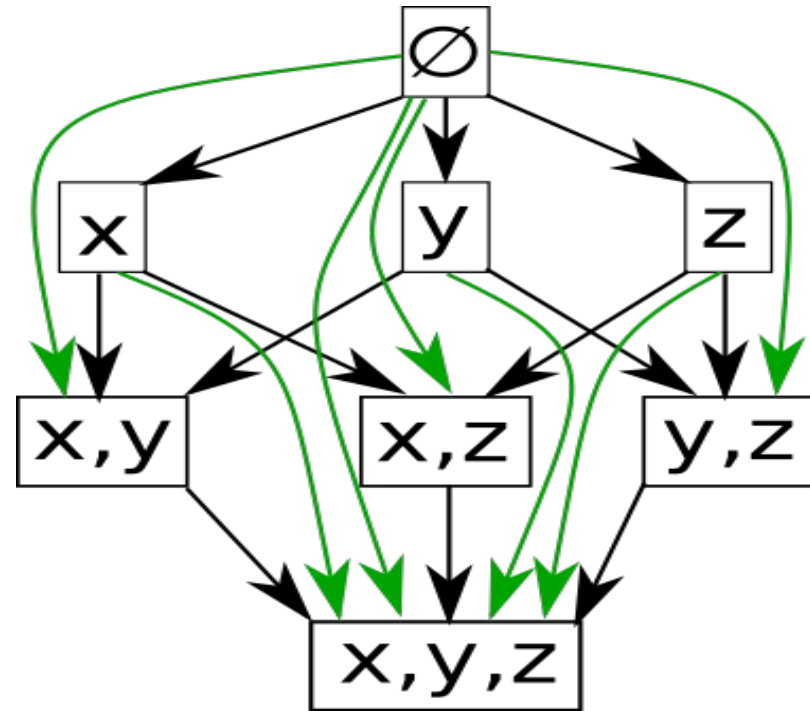
# DAG / LPO

Smerový **a**cyklický **g**raf.

Označené **Č**iastočné **U**sporiadanie.  $(V, \rightarrow)$

Tranzitívna, ireflexívna.

maximálny paralelizmus, probl. maximálneho toku



# DAG / LPO

**Čiastočné usporiadanie** je dvojica  $(V, \rightarrow)$ , kde

$V$  je množina vrcholov/udalostí,

$\rightarrow$  je ireflexívna tranzitívna binárna relácia

Čiastočné usporiadanie spolu s funkciou, ktorá každému prvku z  $V$  priradí nejaký prvok z množiny  $T$  budeme nazývať **označené čiastočné usporiadanie**. **Rezum** v čiastočnom usporiadaní sa nazýva taká podmnožina  $X_s \subseteq V$ , že

$$\forall x, y \in X_s: ((x, y) \notin \rightarrow) \wedge ((y, x) \notin \rightarrow),$$

$$\forall v \in V: v \notin X_s, (v, x) \in \rightarrow \wedge (x, v) \in \rightarrow$$

**Minulosť rezu**  $X_s$  (označované ako  $X_m$ )  $X_m: \{v \in V \mid (\exists x \in X_s: v \rightarrow x)\}$

# WC: Wrong continuations

Pozorované scénare:  $\{a;b;d\}$ ,  $\{a;c;d\}$

Sekvencia  $a;b;d$ :  $m \geq a_z$ ;  $m \geq a_z - a_d + b_z$ ;  $m \geq a_z - a_d + b_z - b_d + d_z$ ;

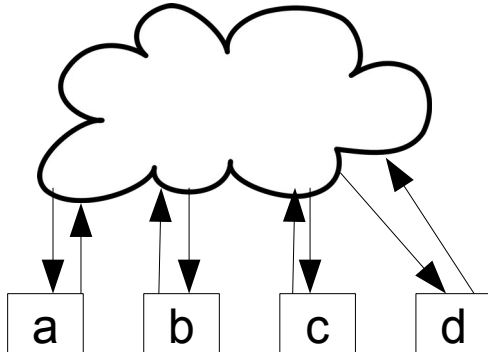
Sekvencia  $a;c;d$ :  $m \geq a_z$ ;  $m \geq a_z - a_d + c_z$ ;  $m \geq a_z - a_d + c_z - c_d + d_z$ ;

Sekvencie  $\{b\}$ ,  $\{c\}$ ,  $\{d\}$ :  $m < b_z$ ;  $m < c_z$ ;  $m < d_z$ ; ďalej

Sekvencie  $\{(a);d\}$ ,  $\{(a);a\}$ :  $m < a_z - a_d + d_z$ ;  $m < 2a_z - a_d$ ;

Sekvencie  $\{(ab);a\}$ ,  $\{(ab);b\}$ ,  $\{(ab);c\}$ ,  $\{(ac);a\}$ ,  $\{(ac);b\}$ ,  $\{(ac);c\}$

Sekvencie  $\{(abd);a\}$ ,  $\{(abd);b\}$ ,  $\{(abd);c\}$ ,  $\{(abd);d\}$ ,  
 $\{(acd);a\}$ ,  $\{(acd);b\}$ ,  $\{(acd);c\}$ ,  $\{(acd);d\}$



$X_m = (a, b, c, d)$ ;

minulost

$X_s = (a, b, c, d)$ ;

spustenie  $\{X_a; X_b; X_c; X_d\}$

$$m^T_m \geq I \cdot X^T_s$$

$$m^T_m = m^T + O \cdot X^T_m - I \cdot X^T_m = m^T \dots +/- \dots$$

$$m^T_a < I \cdot X^T_d$$

$$m^T + O \cdot X^T_a - I \cdot X^T_a < I \cdot X^T_d$$

$$m^T < a_z - a_d + d_z$$

$$m^T_{acd} < I \cdot X^T_a$$

$X_m = (1, 0, 1, 1)$ ;  $X_s = (1, 0, 0, 0)$ ;

$$m^T + O \cdot X^T_{acd} - I \cdot X^T_{acd} < I \cdot X^T_a$$

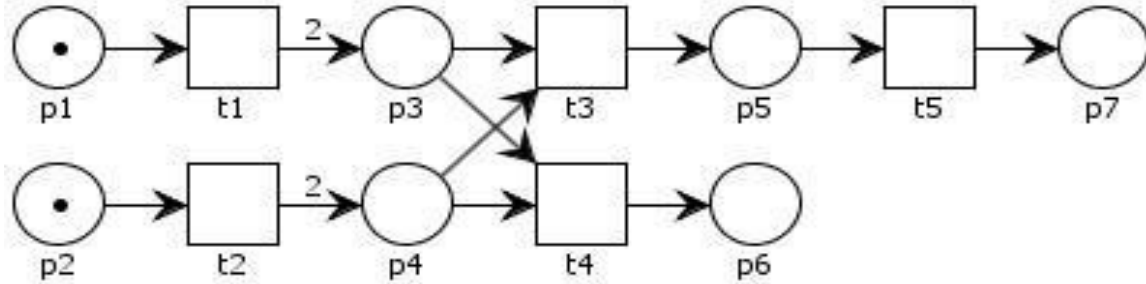
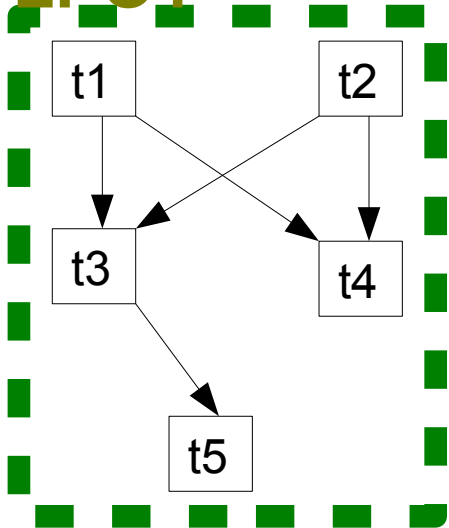
$$m^T < a_z - a_d + c_z - c_d + d_z - d_d + a_z$$

...

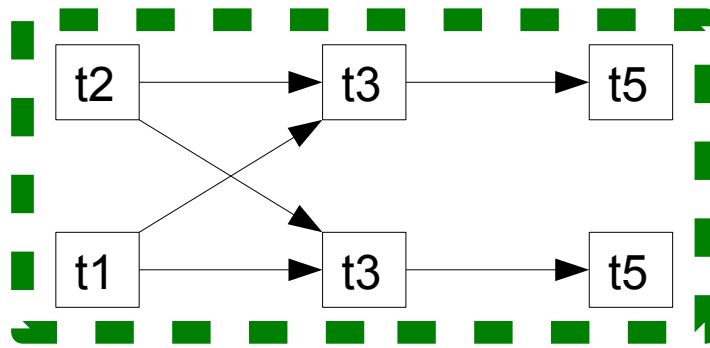
# DAG / LPO

Určite či je LPO spustiteľné v PS.

LPO1



LPO2



# LPO cut

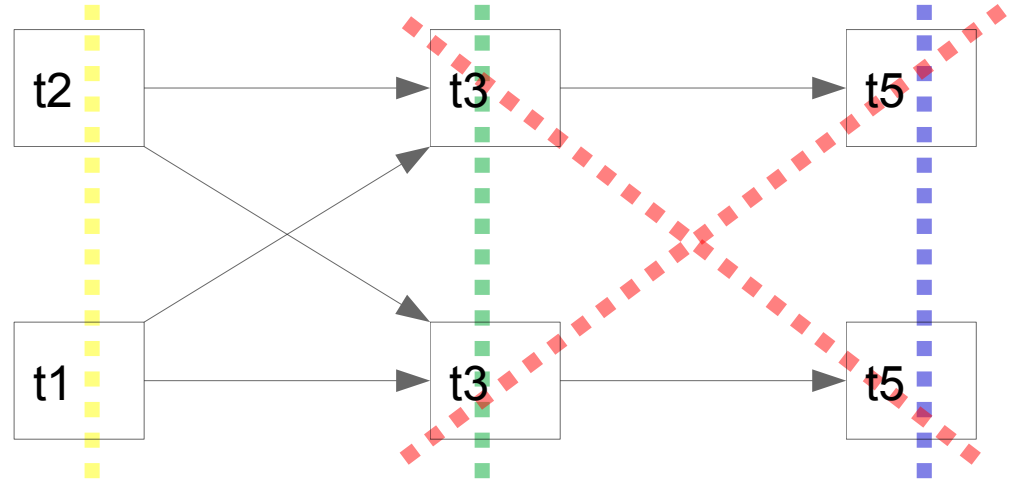
podľa definície rezu:

(t1+t2)

(t3+t3)

(t3+t5)\*

(t5+t5)



d'alšie možné, redundantné kontroly:

t1, t2, t3\*, t5\* -- **nie sú rezy**

$$\forall x,y \in X_s : ((x,y) \notin \rightarrow) \wedge ((y,x) \notin \rightarrow),$$

$$\forall v \in V : v \notin X_s, (v,x) \in \rightarrow \wedge (x,v) \in \rightarrow$$

# LPO cut

$$X_s=(t1+t2):$$

$$X_m=\{\}$$

$$X_s=(t3+t3):$$

$$X_m=\{t1,t2\}$$

$$X_s=(t3+t5):$$

$$X_m=\{t1,t2,t3\}$$

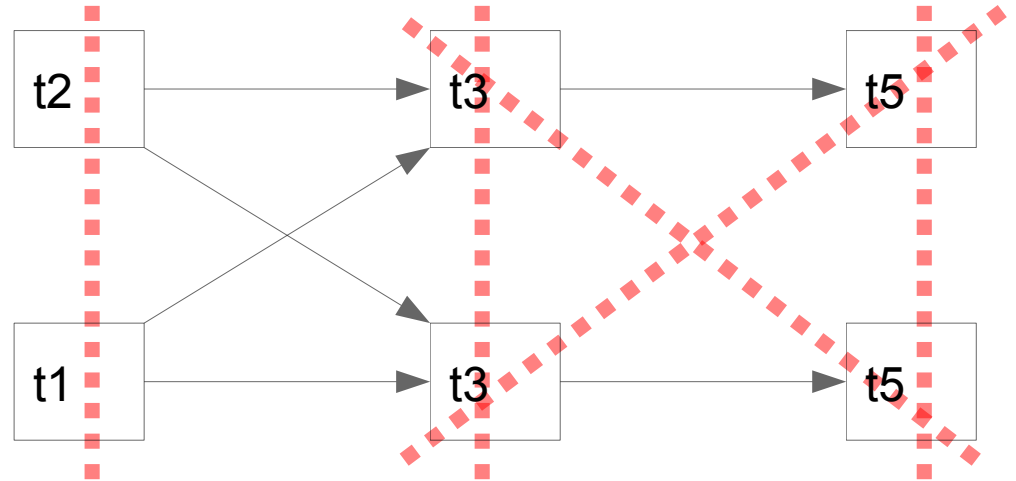
$$X_s=(t5+t5):$$

$$X_m=\{t1,t2,t3,t3\}$$

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$$X_s=(2.t5):$$

$$X_m=\{t1+t2+2.t3\}$$



# executability LPO

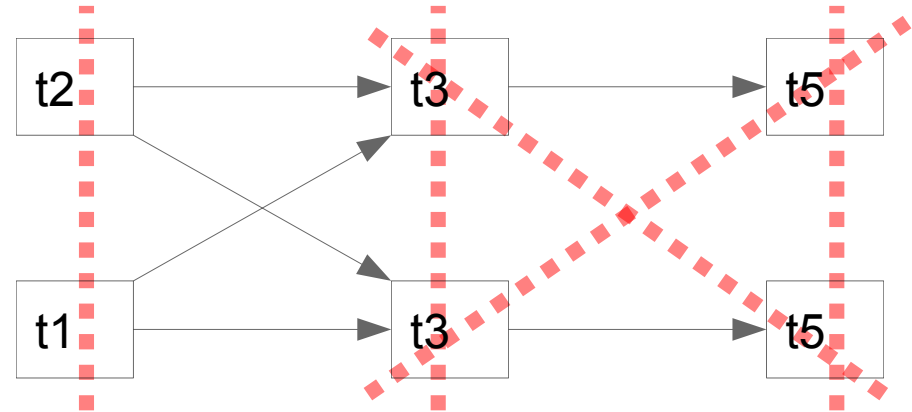
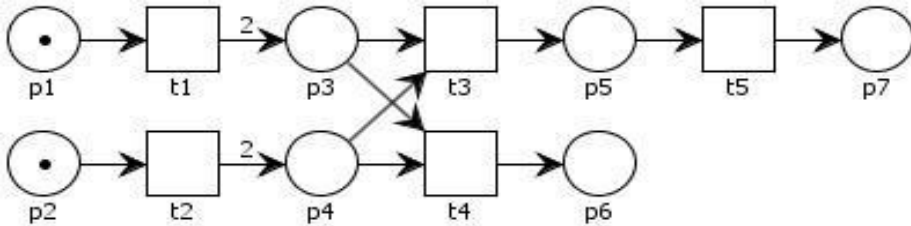
$X_s\{t3+t3\}$ :  $X_m=\{t1,t2\}$

$X_s=(0,0,2,0,0)$

$X_m=(1,1,0,0,0)$

$$m_0^T(p) + \begin{matrix} C(p,t) \\ \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & -1 & 0 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \cdot X_m^T(t) \geq \begin{matrix} I(p,t) \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix} \cdot X_s^T(t)$$

$$m_{dosiahnute} = (0, 0, 2, 2, 0, 0, 0) \geq (0, 0, 2, 2, 0, 0, 0) = m_{nutne}$$





# executability LPO

rezy:

(1-2)

(3)

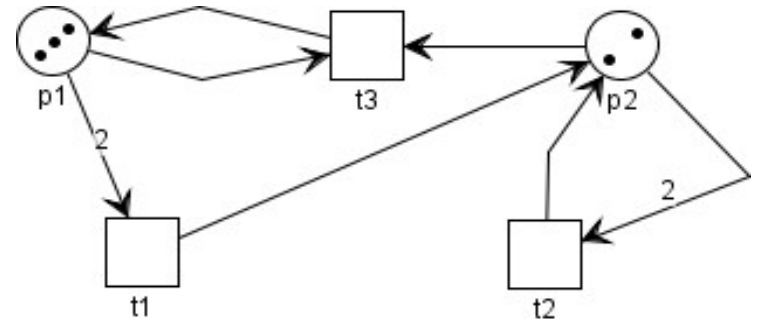
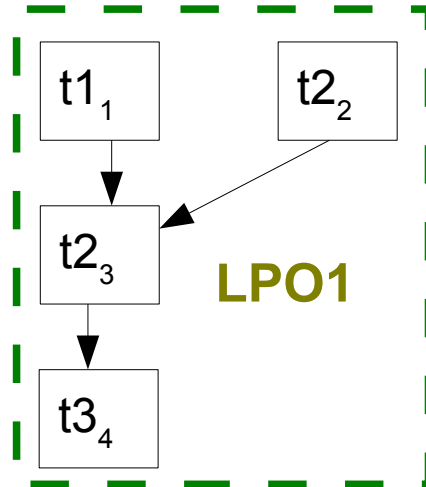
(4)

$$m_0^T(p) + C(p, t) \cdot X_m^T(t) \geq I(p, t) \cdot X_s^T(t)$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 & 0 & 0 \\ 1 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \geq \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

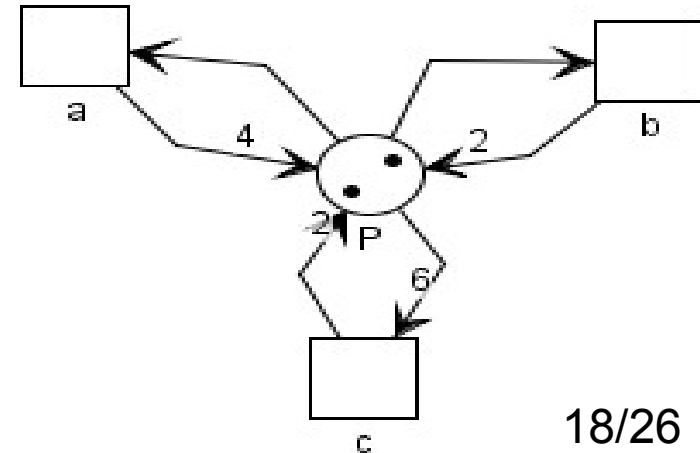
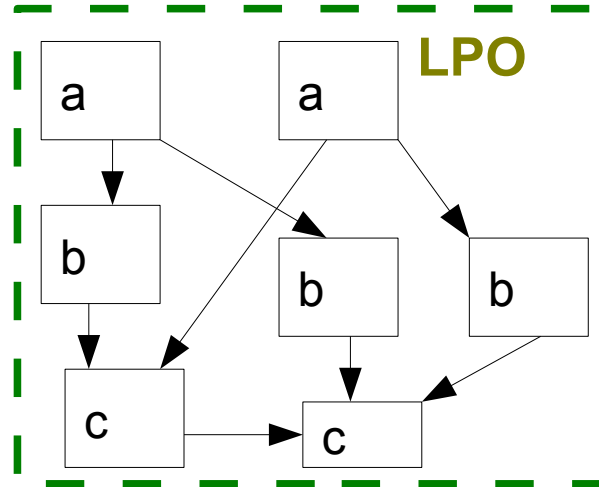
$$m_{dosiahnute} = (1, 1) \geq (1, 1) = m_{nutne}$$



# executability LPO

rez:

minulost rezu:



# executability LPO

rezy:

(a<sub>1</sub>+a<sub>2</sub>)

(a<sub>1</sub>+b<sub>3</sub>)

(a<sub>2</sub>+b<sub>1</sub>+b<sub>2</sub>)

(b+b+b)

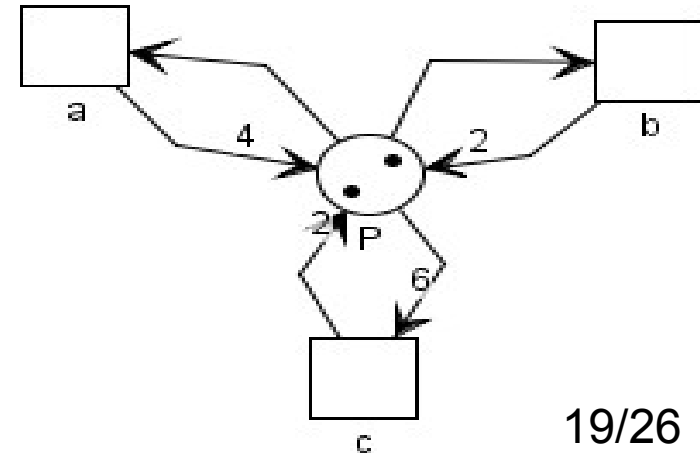
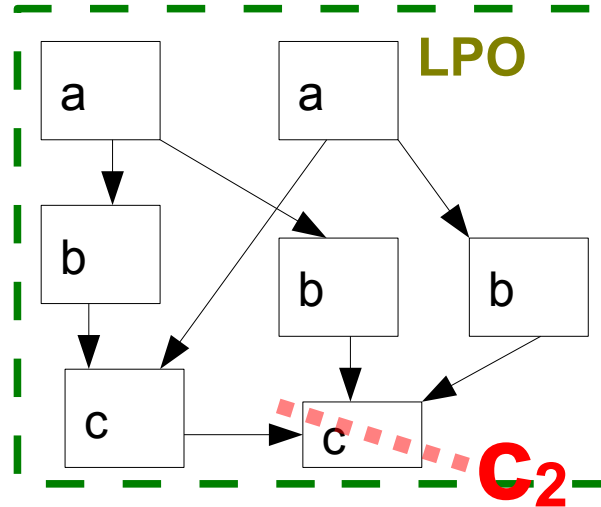
**(b<sub>2</sub>+b<sub>3</sub>+c<sub>1</sub>)**

postačujúce??

$$m_0^T + C \cdot X_m^T \geq I \cdot X_s^T$$

$$(2) + (3 \ 1 \ -4) \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \geq (1 \ 1 \ 6) \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$m_{dosiahnute} = (9) \geq (8) = m_{nutne}$$



# executability LPO

rezy:

$(a_1+a_2)$

$(a_1+b_3)$

$(a_2+b_1+b_2)$

$(b+b+b)$

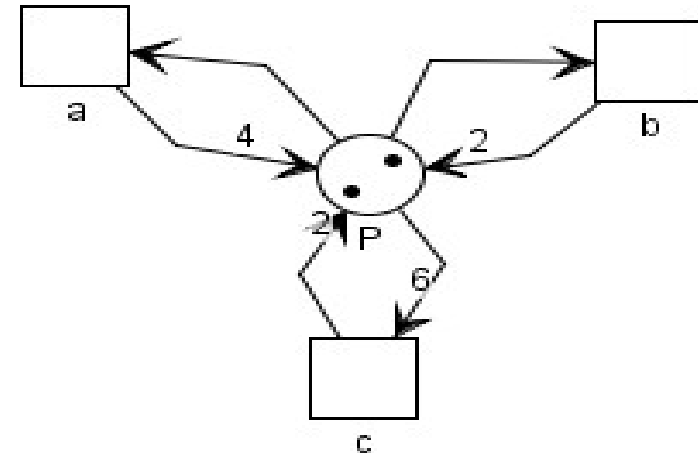
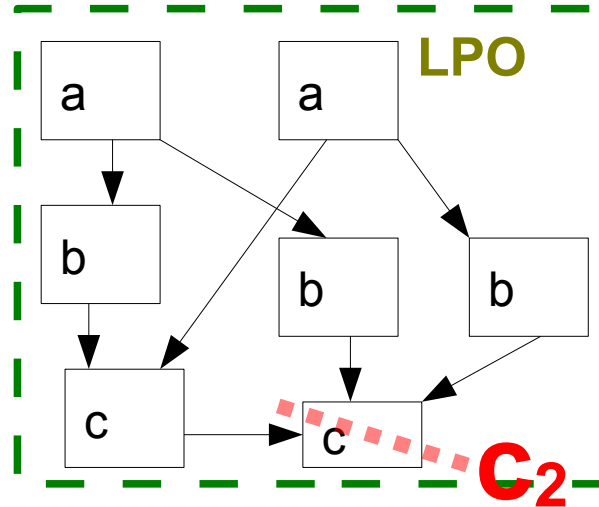
$(b_2+b_3+c_1)$

$C_2$

$$m_0^T + C \cdot X_m^T \geq I \cdot X_s^T$$

$$(2) + (3 \ 1 \ -4) \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \geq (1 \ 1 \ 6) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

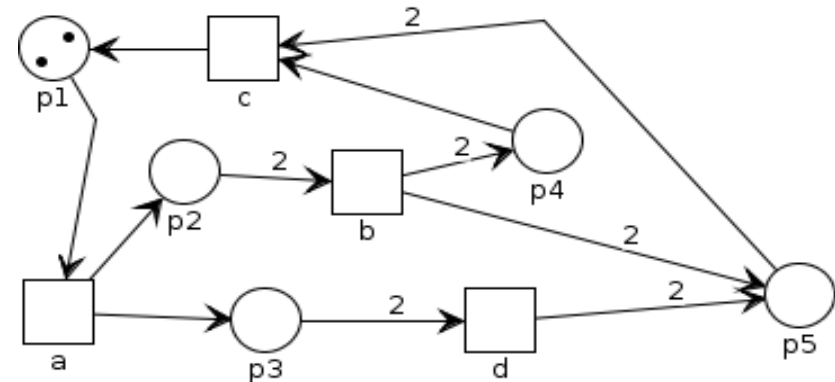
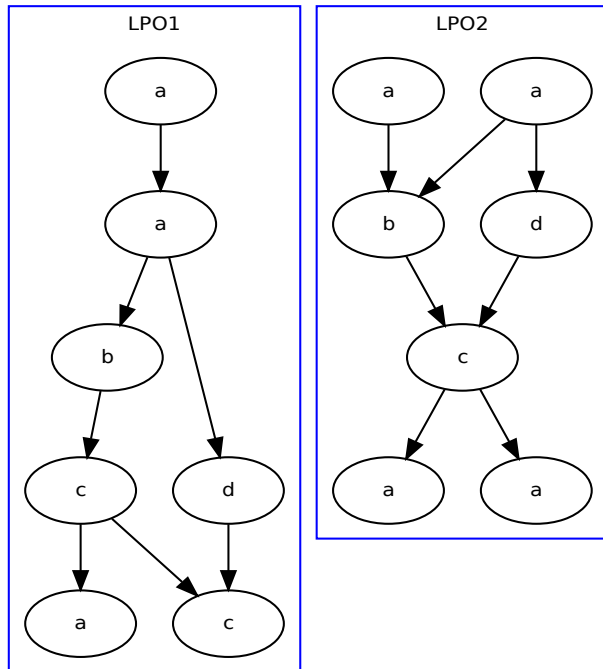
$$m_{dosiahnute} = (7) \geq (6) = m_{nutne}$$



# executability LPO

úloha (test 2017):

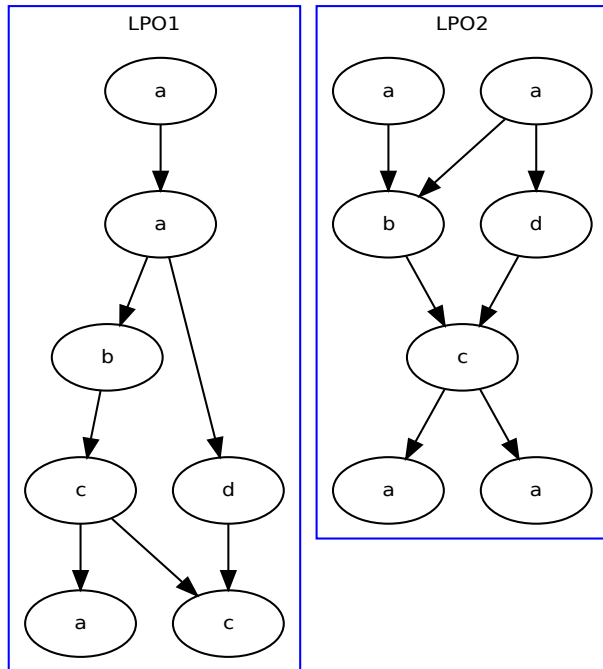
Pre PS a čiastočné usporiadania ( LPO1 LPO2) zistite spustiteľnosť jednotlivých LPO. Ak je LPO spustiteľné vypíšte jeho rezy, ak nie zapíšte ten, pre ktorý nie je spustiteľné.



# executability LPO

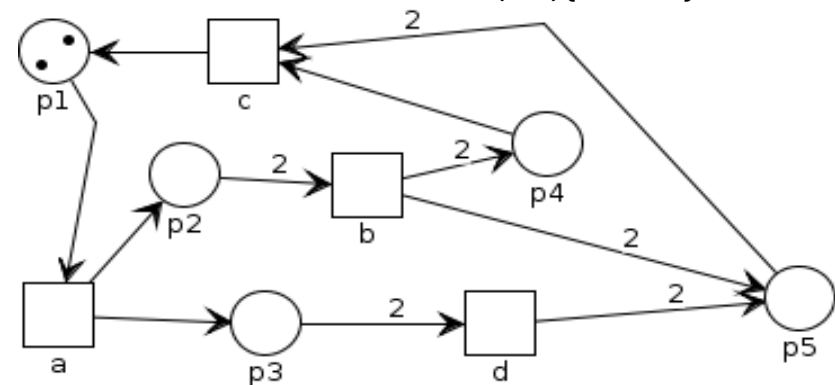
úloha (test 2017):

Pre PS a čiastočné usporiadania ( LPO1 LPO2) zistite spustiteľnosť jednotlivých LPO. Ak je LPO spustiteľné vypíšte jeho rezy, ak nie zapíšte ten, pre ktorý nie je spustiteľné.



LPO1:  
 $v_1(a)\{\};$   
 $v_2(a)\{a\};$   
 $v_3v_5(bd)\{aa\};$   
 $v_4v_5(cd)\{aab\};$   
 $v_5v_6(ad)\{aabc\};$   
 $v_6v_7(ac)\{aabcd\}$

LPO2:  
 $v_1v_2(aa)\{\};$   
 $v_1v_4(ad)\{a\};$   
 $v_3v_4(bd)\{aa\};$   
 $v_5(c)\{aabd\};$   
 $v_6v_7(aa)\{aabcd\}$



# executability LPO

úloha (test 2017):

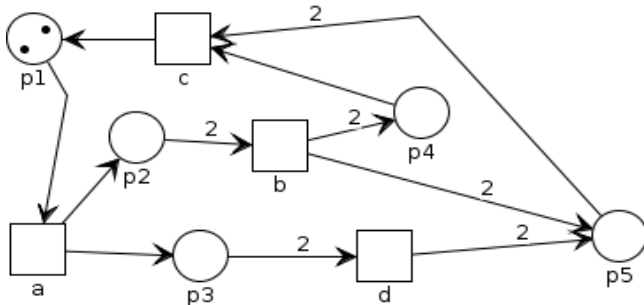
Pre PS a čiastočné usporiadania ( LPO1 LPO2) zistite spustiteľnosť jednotlivých LPO. Ak je LPO spustiteľné vypíšte jeho rezy, ak nie zapíšte ten, pre ktorý nie je spustiteľné.

LPO1:

v1(a){};  
 v2(a){a};  
 v3v5(bd){aa};  
 v4v5(cd){aab};  
 v5v6(ad){aabc};  
 v6v7(ac){aabcd}

LPO2:

v1v2(aa){};  
 v1v4(ad){a};  
 v3v4(bd){aa};  
 v5(c){aabd};  
 v6v7(aa){aabcd}



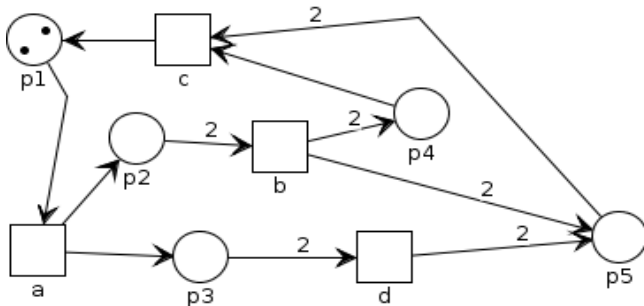
# executability LPO

úloha (test 2017):

Pre PS a čiastočné usporiadania ( LPO1 LPO2) zistite spustiteľnosť jednotlivých LPO. Ak je LPO spustiteľné vypíšte jeho rezy, ak nie zapíšte ten, pre ktorý nie je spustiteľné.

LPO1:  
 v1(a){};  
 v2(a){a};  
 v3v5(bd){aa};  
 v4v5(cd){aab};  
 v5v6(ad){aabc};  
 v6v7(ac){abcd}

LPO2:  
 v1v2(aa){};  
 v1v4(ad){a};  
 v3v4(bd){aa};  
 v5(c){aabd};  
 v6v7(aa){abcd}

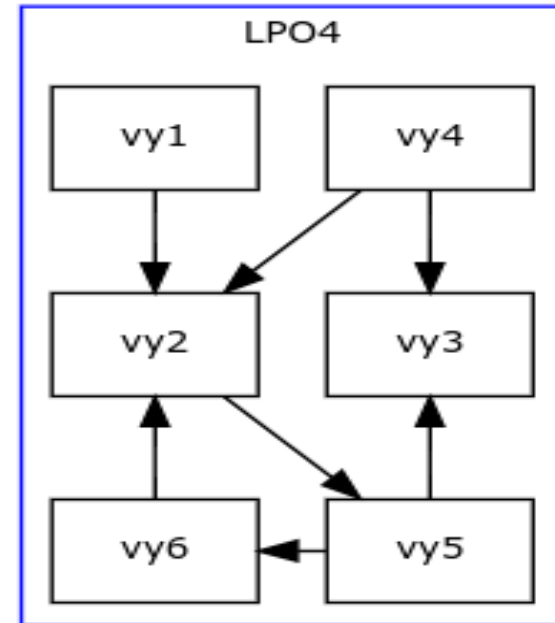
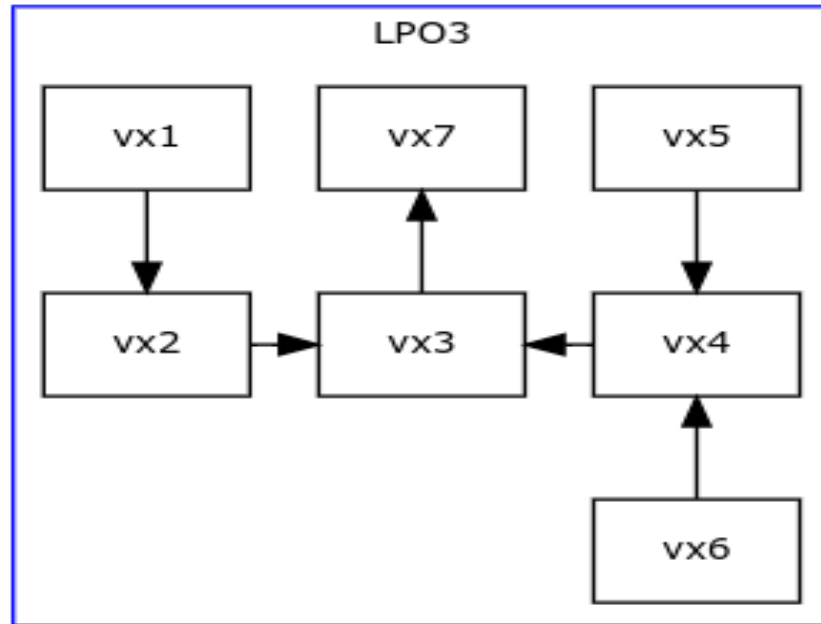


$$\begin{array}{l}
 m_0^T(p) + C(p, t) \cdot X_m^T(t) \geq I(p, t) \cdot X_s^T(t) \\
 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & -2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \geq \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 \hline
 X_m = (0, 0, 0, 0) : (2, 0, 0, 0, 0) \geq (1, 0, 0, 0, 0) : X_s = (1, 0, 0, 0) \\
 X_m = (1, 0, 0, 0) : (1, 1, 1, 0, 0) \geq (1, 0, 0, 0, 0) : X_s = (1, 0, 0, 0) \\
 X_m = (2, 0, 0, 0) : (0, 2, 2, 0, 0) \geq (0, 2, 2, 0, 0) : X_s = (0, 1, 0, 1) \\
 X_m = (2, 1, 0, 0) : (0, 0, 2, 2, 2) \geq (0, 0, 2, 1, 2) : X_s = (0, 0, 1, 1) \\
 X_m = (2, 1, 1, 0) : (1, 0, 2, 1, 0) \geq (1, 0, 2, 0, 0) : X_s = (1, 0, 0, 1) \\
 X_m = (2, 1, 1, 1) : (1, 0, 0, 1, 2) \geq (1, 0, 0, 1, 2) : X_s = (1, 0, 1, 0) \\
 \hline
 X_m = (0, 0, 0, 0) : (2, 0, 0, 0, 0) \geq (2, 0, 0, 0, 0) : X_s = (2, 0, 0, 0) \\
 X_m = (1, 0, 0, 0) : (1, 1, 1, 0, 0) \geq (1, 0, 2, 0, 0) : X_s = (1, 0, 0, 1) \\
 X_m = (2, 0, 0, 0) : (0, 2, 2, 0, 0) \geq (0, 2, 2, 0, 0) : X_s = (0, 1, 0, 1) \\
 X_m = (2, 0, 1, 1) : (1, 2, 0, -1, 0) \geq (0, 0, 0, 1, 2) : X_s = (0, 0, 1, 0) \\
 X_m = (2, 1, 1, 1) : (1, 0, 0, 1, 2) \geq (2, 0, 0, 0, 0) : X_s = (2, 0, 0, 0)
 \end{array}$$



# executability LPO

Vypíšte rezy pre čiastočné usporiadania ( LPO3 LPO4). Vypíšte pre jednotlivé rezy ich minulosť.



# executability LPO

Vypíšte rezy pre čiastočné usporiadania ( LPO3 LPO4). Vypíšte pre jednotlivé rezy ich minulosť.

LPO3:

$vx1, vx5, vx6\}$

$vx1, vx4\{vx5, vx6\}$

$vx2, vx5, vx6\{vx1\}$

$vx2, vx4\{vx1, vx5, vx6\}$

$vx3\{vx1, vx2, vx4, vx5, vx6\}$

$vx7\{vx1, vx2, vx3, vx4, vx5, vx6\}$

LPO4: nie je DAG

$vy5, vy6, vy2$

