

Úlohy:

1. K daným gramatikám nájdite množiny *FIRST*, *FOLLOW* pre neterminálne symboly a *FIRST* pre reťazce α .

- (a) $S \rightarrow AB$
 $A \rightarrow BA \mid a$
 $B \rightarrow bB \mid Aa \mid \varepsilon$
 $\alpha = AA, \alpha = BA, \alpha = BB, \alpha = bAA, \alpha = BaB$

Riešenie:

	<i>S</i>	<i>A</i>	<i>B</i>
<i>FIRST</i>	{ <i>a, b</i> }	{ <i>a, b</i> }	{ ε, a, b }
<i>FOLLOW</i>	{ ε }	{ ε, a, b }	{ ε, a, b }

- $FIRST(AA) = \{a, b\}$
 - $FIRST(BA) = \{a, b\}$
 - $FIRST(BB) = \{\varepsilon, a, b\}$
 - $FIRST(bAA) = \{b\}$
 - $FIRST(BaB) = \{a, b\}$
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- (b) $S \rightarrow bAa$
 $A \rightarrow BB \mid cB$
 $B \rightarrow aBa \mid \varepsilon$
 $\alpha = AB, \alpha = BA, \alpha = AA, \alpha = BB$

Riešenie:

	<i>S</i>	<i>A</i>	<i>B</i>
<i>FIRST</i>	{ <i>b</i> }	{ ε, a, c }	{ ε, a }
<i>FOLLOW</i>	{ ε }	{ <i>a</i> }	{ <i>a</i> }

- $FIRST(AB) = \{a, c, \varepsilon\}$
 - $FIRST(BA) = \{a, c, \varepsilon\}$
 - $FIRST(AA) = \{a, c, \varepsilon\}$
 - $FIRST(BB) = \{a, \varepsilon\}$
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- (c) $S \rightarrow AaB$
 $A \rightarrow BbS \mid a$
 $B \rightarrow SA \mid b$
 $\alpha = AB, \alpha = BA, \alpha = AA, \alpha = BB$

Riešenie:

	S	A	B
$FIRST$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$
$FOLLOW$	$\{\varepsilon, a, b\}$	$\{\varepsilon, a, b\}$	$\{\varepsilon, a, b\}$

- $FIRST(AB) = \{a, b\}$
- $FIRST(BA) = \{a, b\}$
- $FIRST(AA) = \{a, b\}$
- $FIRST(BB) = \{a, b\}$

- (d) $S \rightarrow aSb \mid bAa \mid Aa$
 $A \rightarrow aS \mid SBa$
 $B \rightarrow Ab \mid b$
 $\alpha = AB, \alpha = BA, \alpha = AA, \alpha = BB$

Riešenie: Gramatika nie je redukovaná!!! Počas jej redukcie zistíme, že množina $N_T = \{B\}$, teda neobsahuje počiatkový neterminál a gramatika generuje iba prázdny jazyk. V takom prípade nemá určovanie $FIRST$ a $FOLLOW$ zmysel.

- (e) $S \rightarrow ABCA$
 $A \rightarrow AB \mid \varepsilon$
 $B \rightarrow AC \mid a$
 $C \rightarrow BA \mid b \mid \varepsilon$
 $\alpha = AB, \alpha = BA, \alpha = AA, \alpha = BB$

Riešenie:

	S	A	B	C
$FIRST$	$\{a, b, \varepsilon\}$	$\{a, b, \varepsilon\}$	$\{a, b, \varepsilon\}$	$\{a, b, \varepsilon\}$
$FOLLOW$	$\{\varepsilon\}$	$\{a, b, \varepsilon\}$	$\{a, b, \varepsilon\}$	$\{a, b, \varepsilon\}$

- $FIRST(AB) = \{a, b, \varepsilon\}$
- $FIRST(BA) = \{a, b, \varepsilon\}$
- $FIRST(AA) = \{a, b, \varepsilon\}$
- $FIRST(BB) = \{a, b, \varepsilon\}$

- (f) E je počiatočný neterminál
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \mathbf{id}$
 $\alpha = E + T, \alpha = T + E, \alpha = TT, \alpha = T * T$

Riešenie:

	E	T	F
<i>FIRST</i>	$\{(\mathbf{id})\}$	$\{(\mathbf{id})\}$	$\{(\mathbf{id})\}$
<i>FOLLOW</i>	$\{\varepsilon, +,)\}$	$\{*, \varepsilon, +,)\}$	$\{*, \varepsilon, +,)\}$

- $FIRST(E + T) = \{(\mathbf{id})\}$
- $FIRST(T + E) = \{(\mathbf{id})\}$
- $FIRST(TT) = \{(\mathbf{id})\}$
- $FIRST(T * T) = \{(\mathbf{id})\}$

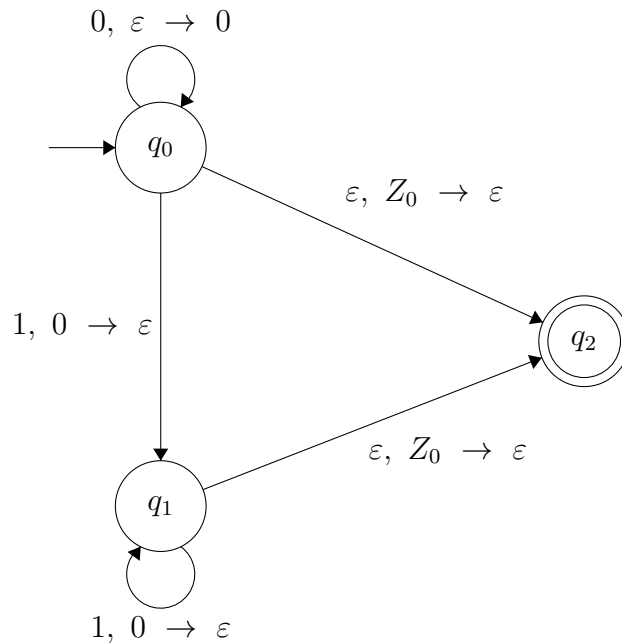
- (g) E je počiatočný neterminál
 $E \rightarrow T \mid T\acute{E}$
 $\acute{E} \rightarrow +T \mid +T\acute{E}$
 $T \rightarrow F \mid F\acute{T}$
 $\acute{T} \rightarrow *F \mid *F\acute{T}$
 $F \rightarrow (E) \mid \mathbf{id}$
 $\alpha = E + T, \alpha = +ET, \alpha = TT, \alpha = *F * F$

Riešenie:

	E	\acute{E}	T	\acute{T}	F
<i>FIRST</i>	$\{(\mathbf{id})\}$	$\{+\}$	$\{(\mathbf{id})\}$	$\{*\}$	$\{(\mathbf{id})\}$
<i>FOLLOW</i>	$\{\varepsilon,)\}$	$\{\varepsilon,)\}$	$\{\varepsilon, +,)\}$	$\{\varepsilon, +,)\}$	$\{\varepsilon, *, +,)\}$

- $FIRST(E + T) = \{(\mathbf{id})\}$
- $FIRST(+ET) = \{+\}$
- $FIRST(TT) = \{(\mathbf{id})\}$
- $FIRST(*F * F) = \{*\}$

2. Simulujte činnosť zásobníkového automatu pre slová w .



$w \in \{\varepsilon, 0, 1, 01, 10\}$.

Aký jazyk akceptuje uvedený zásobníkový automat?

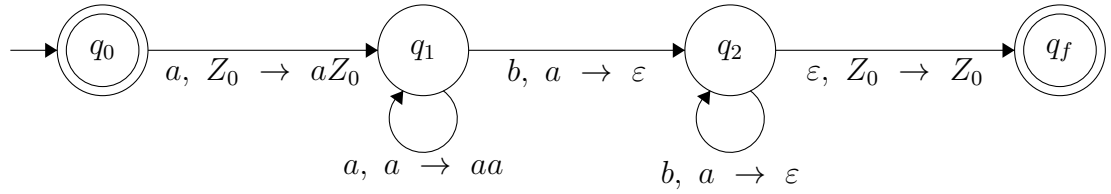
Riešenie:

- $w = \varepsilon$:
 - $(q_0, \varepsilon, Z_0) \vdash (q_2, \varepsilon, \varepsilon)$, slovo ε akceptuje.
- $w = 0$:
 - $(q_0, 0, Z_0) \vdash (q_2, 0, \varepsilon) \vdash \text{ZÁSEK}$
 - $(q_0, 0, Z_0) \vdash (q_0, \varepsilon, 0Z_0) \vdash \text{ZÁSEK}$
 - Iné výpočty neexistujú, slovo 0 neakceptuje.
- $w = 1$:
 - $(q_0, 1, Z_0) \vdash (q_2, 1, \varepsilon) \vdash \text{ZÁSEK}$
 - Iné výpočty neexistujú, slovo 1 neakceptuje.
- $w = 01$:
 - $(q_0, 01, Z_0) \vdash (q_2, 01, \varepsilon) \vdash \text{ZÁSEK}$
 - $(q_0, 01, Z_0) \vdash (q_0, 1, 0Z_0) \vdash (q_1, \varepsilon, Z_0) \vdash (q_2, \varepsilon, \varepsilon)$, slovo 01 akceptuje.
- $w = 10$:
 - $(q_0, 10, Z_0) \vdash (q_2, 10, \varepsilon) \vdash \text{ZÁSEK}$
 - Iné výpočty neexistujú, slovo 10 neakceptuje.
- Automat rozpoznáva jazyk $L = \{0^n 1^n \mid n \in \mathbb{Z}, n \geq 0\}$

3. Nájdnite zásobníkový automat, ktorý rozpoznáva jazyk

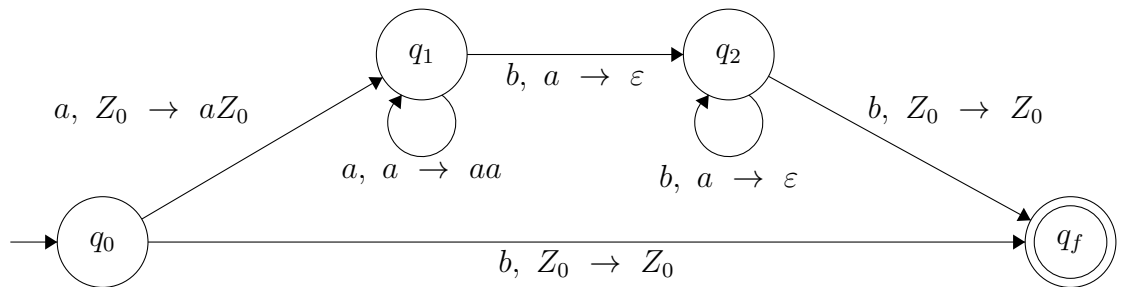
(a) $L_1 = \{a^n b^n \mid n \in \{0, 1, 2, \dots\}\}$

Riešenie:



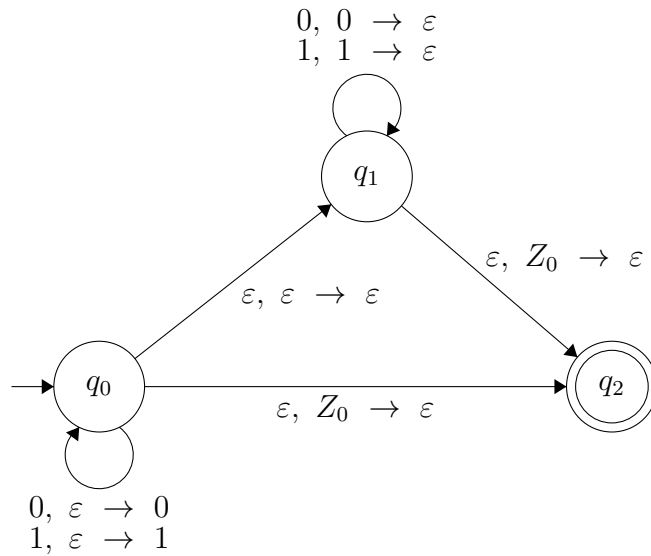
(b) $L_2 = \{a^n b^{n+1} \mid n \in \{0, 1, 2, \dots\}\}$

Riešenie:



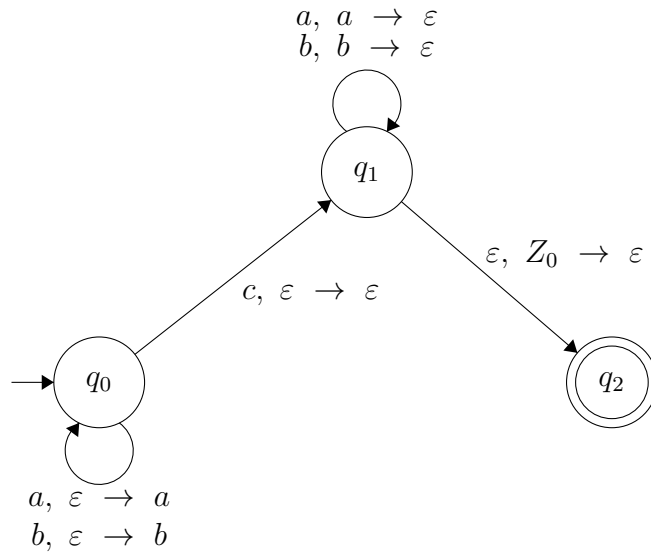
(c) $L_3 = \{ww^R \mid w \in \{0, 1\}^*\}$

Riešenie:



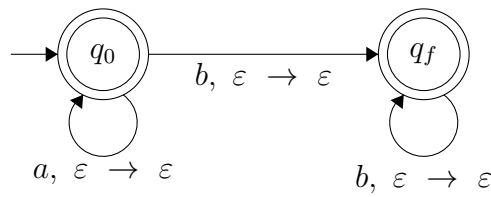
(d) $L_4 = \{wcw^R \mid w \in \{a, b\}^*\}$

Riešenie:



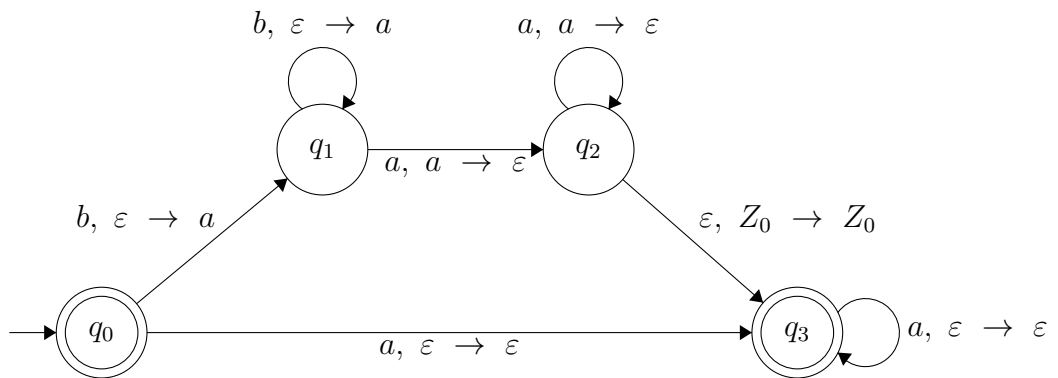
(e) $L_5 = \{a^*b^*\}$

Riešenie:



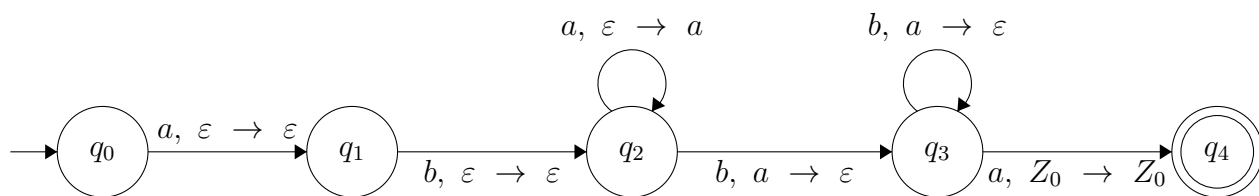
(f) $L_6 = \{b^n a^{n+m} \mid n \in \{0, 1, 2, \dots\}, m \in \{0, 1, 2, \dots\}\}$

Riešenie:



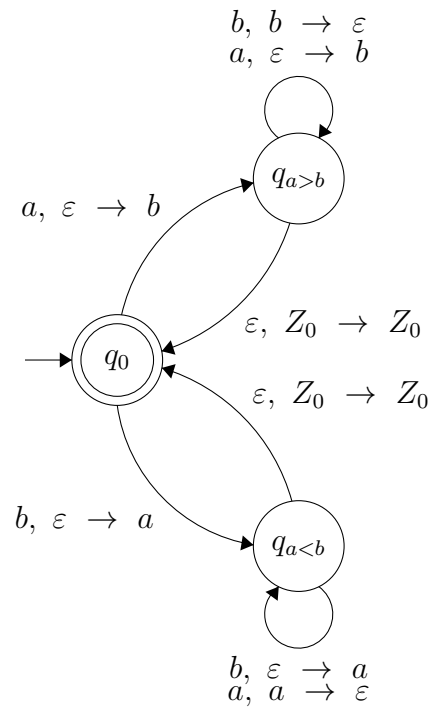
(g) $L_7 = \{aba^n b^n a \mid n \in \{1, 2, \dots\}\}$

Riešenie:



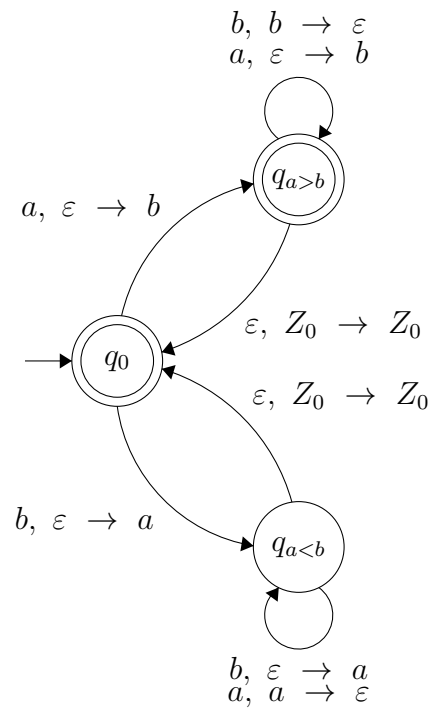
(h) $L_8 = \{w \mid w \in \{a, b\}^*, \#_a(w) = \#_b(w)\}$

Riešenie:



(i) $L_9 = \{w \mid w \in \{a, b\}^*, \#_a(w) \geq \#_b(w)\}$

Riešenie:



(j) $L_{10} = \{w \mid w \in \{a, b\}^*, \#_a(w) \leq \#_b(w)\}$

Riešenie:

