

Cvičenie č. 3

Úlohy na cvičenie:

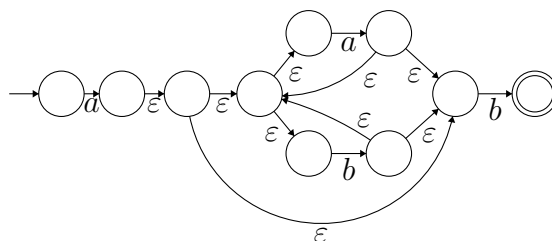
1. Nájdite NKA (príp. DKA), ktorý rozpoznáva jazyk daný regulárnym výrazom:

- (a) $a(a | b)^*b$
- (b) $(ab)^*|(ba)^*$
- (c) $a | (bc)^*d$
- (d) $(0 | 1)^*(2 | 3)(2 | 3)^*$
- (e) (begin | end | while | with)
- (f) $(a^* | b^*)^*$

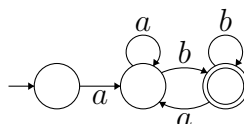
Riešenia:

- (a) $a(a | b)^*b$

(NKA ako výsledok Thompsonovej konštrukcie:)

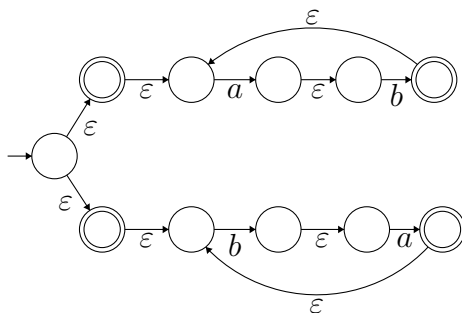


alebo DKA:

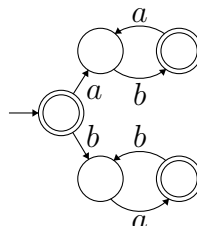


- (b) $(ab)^* | (ba)^*$

(NKA ako výsledok Thompsonovej konštrukcie:)

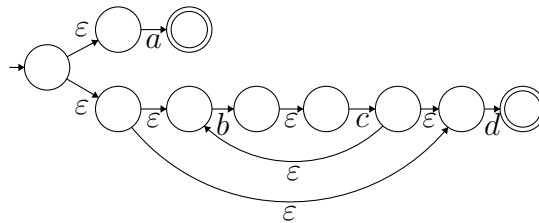


alebo DKA:

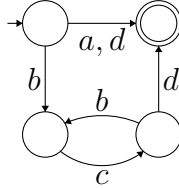


(c) $a \mid (bc)^*d$

(NKA ako výsledok Thompsonovej konštrukcie:)

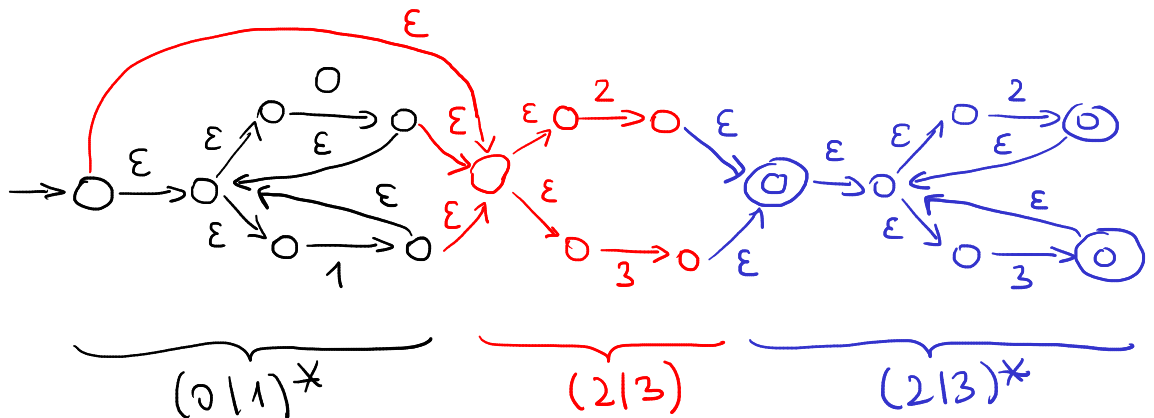


alebo DKA:

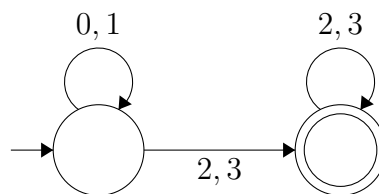


(d) $(0 \mid 1)^*(2 \mid 3)(2 \mid 3)^*$

(NKA ako výsledok Thompsonovej konštrukcie:)

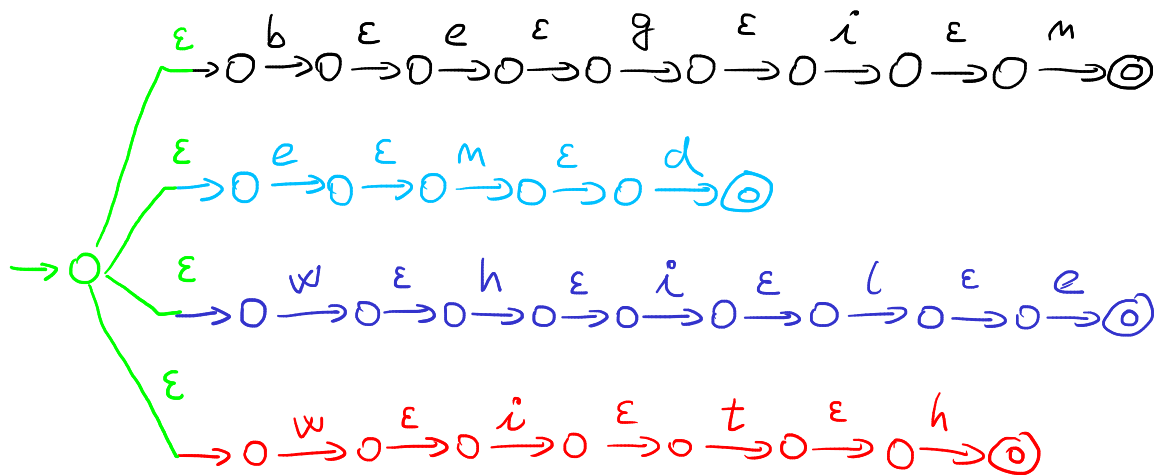


alebo DKA:

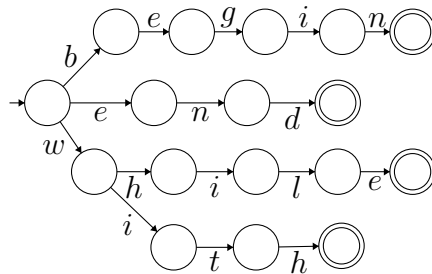


(e) $(\text{begin} \mid \text{end} \mid \text{while} \mid \text{with})$

(NKA ako výsledok Thompsonovej konštrukcie:)

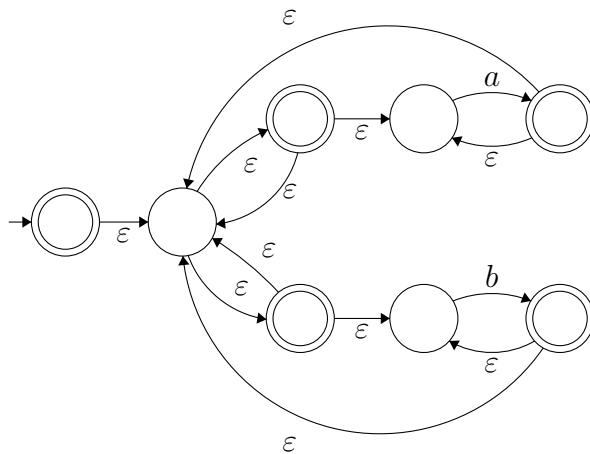


alebo DKA:



(f) $(a^* | b^*)^*$

(NKA ako výsledok Thompsonovej konštrukcie:)



alebo DKA (po uvedení si, že daný jazyk je to isté ako $(a | b)^*$):



2. Ku regulárnym výrazom z úlohy 1 nájdite regulárne gramatiky, ktoré generujú jazyk popísaný danými regulárnymi výrazmi.

Riešenia:

- (a) Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aA \mid bA \mid b \end{aligned}$$

- (b) Gramatika $G = (\{S, A, B\}, \{a, b\}, P, S)$, pravidlá:

$$\begin{aligned} S &\rightarrow abA \mid baB \mid \varepsilon \\ A &\rightarrow abA \mid \varepsilon \\ B &\rightarrow baB \mid \varepsilon \end{aligned}$$

- (c) Gramatika $G = (\{S, A\}, \{a, b, c, d\}, P, S)$, pravidlá:

$$\begin{aligned} S &\rightarrow a \mid d \mid bcA \\ A &\rightarrow bcA \mid d \end{aligned}$$

- (d) Gramatika $G = (\{S, A\}, \{0, 1, 2, 3\}, P, S)$, pravidlá:

$$\begin{aligned} S &\rightarrow 0S \mid 1S \mid 2A \mid 3A \\ A &\rightarrow 2A \mid 3A \mid \varepsilon \end{aligned}$$

- (e) Gramatika $G = (\{S\}, \{b, d, e, g, h, i, l, n, t, w\}, P, S)$, pravidlá:

$$S \rightarrow \text{begin} \mid \text{end} \mid \text{while} \mid \text{with}$$

- (f) Gramatika $G = (\{S\}, \{a, b\}, P, S)$, pravidlá:

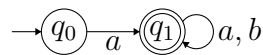
$$S \rightarrow aS \mid bS \mid \varepsilon$$

3. Nasledujúce jazyky popíšte DKA, regulárnou gramatikou a regulárnym výrazom.

$$L_1 = \{aw \mid w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

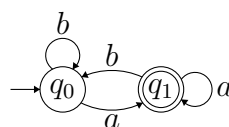
$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aA \mid bA \mid \varepsilon \end{aligned}$$

Regex: $R = a(a \mid b)^*$

$$L_2 = \{wa \mid w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S\}, \{a, b\}, P, S)$, pravidlá:

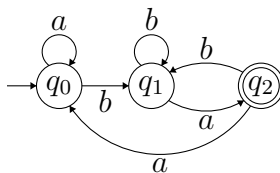
$$S \rightarrow aS \mid bS \mid a$$

Regex: $R = (a \mid b)^*a$

$$L_3 = \{wba \mid w \in \{a, b\}^*\}$$

Riešenie:

DKA



Gramatika $G = (\{S\}, \{a, b\}, P, S)$, pravidlá:

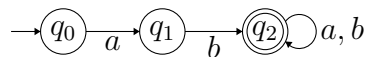
$$S \rightarrow aS \mid bS \mid ba$$

Regex: $R = (a \mid b)^*ba$

$$L_4 = \{abw \mid w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow abA$$

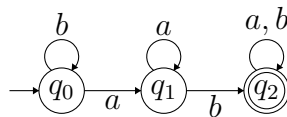
$$A \rightarrow aA \mid bA \mid \varepsilon$$

Regex: $R = ab(a \mid b)^*$

$$L_5 = \{xaby \mid x, y \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow aS \mid bS \mid abA$$

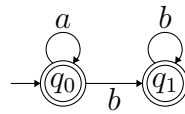
$$A \rightarrow aA \mid bA \mid \varepsilon$$

Regex: $R = (a \mid b)^*ab(a \mid b)^*$

$$L_7 = \{a^*b^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow \varepsilon \mid aS \mid bA$$

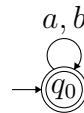
$$A \rightarrow bA \mid \varepsilon$$

Regex: $R = a^*b^*$

$$L_8 = \{a, b\}^*$$

Riešenie:

DKA:



Gramatika $G = (\{S\}, \{a, b\}, P, S)$, pravidlá:

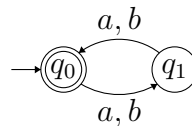
$$S \rightarrow aS \mid bS \mid \varepsilon$$

Regex: $R = (a \mid b)^*$

$$L_9 = \{w \mid \#_a(w) \equiv \#_b(w) \pmod{2}, w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow aA \mid bA \mid \varepsilon$$

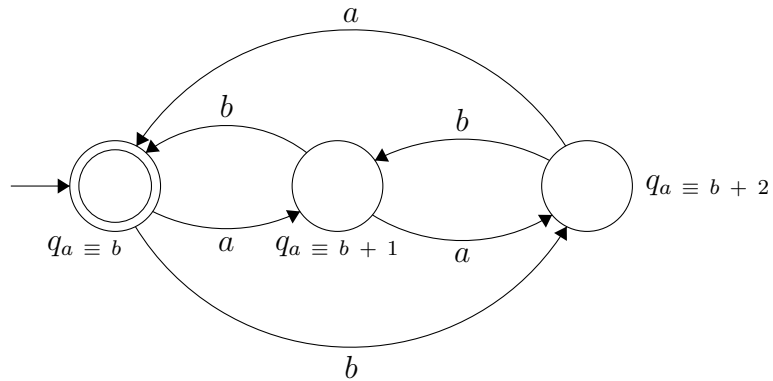
$$A \rightarrow aS \mid bS$$

Regex: $R = ((a \mid b)(a \mid b))^*$

$$L_{10} = \{w \mid \#_a(w) \equiv \#_b(w) \pmod{3}, w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A, B\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow aA \mid bB \mid \varepsilon$$

$$A \rightarrow aB \mid bS$$

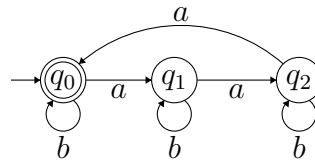
$$B \rightarrow aS \mid bA$$

Regex: *nemusíte vedieť urobiť, je zbytočne zložitý...*

$$L_{11} = \{w \mid \#_a(w) \equiv 0 \pmod{3}, w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A, B\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow bS \mid aA \mid \varepsilon$$

$$A \rightarrow aB \mid bA$$

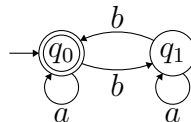
$$B \rightarrow aS \mid bB$$

Regex: $R = (b \mid (ab^*ab^*a))^*$

$$L_{12} = \{w \mid w \text{ má párny počet znakov } b, w \in \{a, b\}^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow aS \mid bA \mid \varepsilon$$

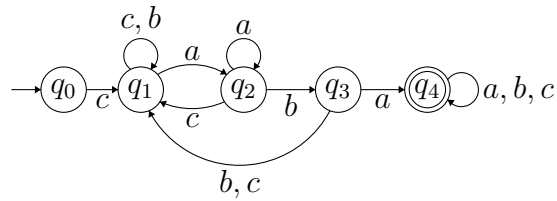
$$A \rightarrow aA \mid bS$$

Regex: $R = (a \mid (ba^*b))^*$

$L_{13} = \{w \mid w \text{ obsahuje ako podreťazec } aba \text{ a začína znakom } c, w \in \{a, b, c\}^*\}$

Riešenie:

DKA:



Gramatika $G = (\{S, A, B\}, \{a, b, c\}, P, S)$, pravidlá:

$$S \rightarrow cA$$

$$A \rightarrow aA \mid bA \mid cA \mid abaB$$

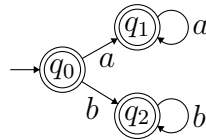
$$B \rightarrow aB \mid bB \mid cB \mid \varepsilon$$

Regex: $R = c(a \mid b \mid c)^*aba(a \mid b \mid c)^*$

$L_{14} = \{a^*b^*\} \cap \{b^*a^*\}$

Riešenie: v podstate ide len o reťazce a^* alebo b^*

DKA:



Gramatika $G = (\{S, A, B\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow aA \mid bB \mid \varepsilon$$

$$A \rightarrow aA \mid \varepsilon$$

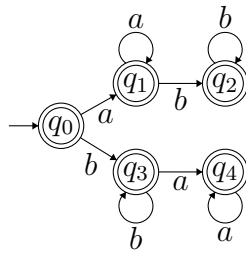
$$B \rightarrow bB \mid \varepsilon$$

Regex: $R = (a^*) \mid (b^*)$

$$L_{15} = \{a^*b^*\} \cup \{b^*a^*\}$$

Riešenie:

DKA:



Gramatika $G = (\{S, A, B, C, D\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow \varepsilon \mid aA \mid bC$$

$$A \rightarrow aA \mid bB \mid \varepsilon$$

$$B \rightarrow bB \mid \varepsilon$$

$$C \rightarrow bC \mid aD \mid \varepsilon$$

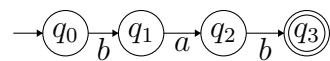
$$D \rightarrow aD \mid \varepsilon$$

Regex: $R = (a^*b^*) \mid (b^*a^*)$

$$L_{16} = \{b^*ab^*\} \cap \{a^*bab\}$$

Riešenie: v podstate ide len o reťazec bab

DKA:



Gramatika $G = (\{S\}, \{a, b\}, P, S)$, pravidlá:

$$S \rightarrow bab$$

Regex: $R = bab$